1 Introduction

The transportation sector is indispensable for economic growth and social development. With both people and goods covering larger distances than ever before, the sector has witnessed a newfound and growing interest by policymakers. In many transport markets interactions between carriers and customers occur in a decentralized manner. This is for instance the case in the markets for taxis, trucks and bulk shipping among others. In these markets, search frictions may result in unrealized trade, thus posing the question search frictions. This comparison allows us to identify the different types of externalities that can result in this setting and derive conditions for each one to be internalized.

We show that search frictions create two types of externalities. First, as is well-known, they generate thin/thick market externalities: when choosing whether to search, agents affect the matching probabilities faced by other agents both in the same and in the opposite side of the market. If agents' search decisions do not internalize this effect, the overall number of agents searching may be distorted away from the efficient one.

Thin/thick market externalities are internalized in equilibrium if and only if the private returns from searching are equal to the social returns. This amounts to the so-called Hosios (1990) conditions on surplus sharing: these conditions, which are well-known to characterize efficiency in search models of labor markets with homogeneous workers, require the share of the surplus which is appropriated by agents on each side of the market to be equal to the elasticity of the matching function with respect to the same side.

Second, search frictions generate what we call pooling externalities: a carrier needs to restart its search once it has dropped off the customer at their destination; however customers may fail to internalize the impact of their destination choice on the distribution of carriers over space. Hence the composition of customers searching for transport to different destinations, and thus the composition of trips realized, may be distorted away from the efficient one.

Customers internalize pooling externalities in equilibrium if and only if prices are such that, carriers receive the same surplus regardless of the customer they match with. This condition for efficiency replicates the no-arbitrage condition obtained in a frictionless world, where competition among carriers ensures that prices coincide with the opportunity cost of each trip, until in equilibrium carriers are indifferent among different customers. In our frictional setup, separate markets for each customer type (e.g. for each destination) are missing: if carriers could compete for a specific customer type, so that heterogeneous customers were not pooled together, in equilibrium carriers would be indifferent across customers. Absent this condition, the price paid by customers for a trip does not reflect its social value and the share of destinations with high social value is too low in equilibrium.

The two efficiency conditions combined characterize analytically the efficient pricing rule, which is useful if a central authority is able to set prices, as in the case of taxicabs. In many markets, however, the planner is not able to directly control prices, but he may be able to impose taxes or subsidies. We show that, when prices are set via Nash bargaining, the planner can achieve efficiency using these instruments and we derive their optimal values. We consider a tax on searching carriers, a tax on searching customers and a tax on trips. The search tax (on either side) is set to equate the private value of an additional agent searching to its social value and forces agents to internalize the thin/thick market externalities. Taxes on trips are used to target the pooling externalities. The optimal trip tax depends on the deviation of the trip's social surplus from the average social surplus across destinations, so that a customer entering a route with social surplus higher (lower) than the average is subsidized (taxed). The planner can restore efficiency by taxing trips and one of the two searching sides.

nd that a destination-speci c tax (customs tax) performs relatively well, as it can achieve 44% of welfare gains achieved under the optimal taxes. In contrast, a tax that is a function of distance achieves no welfare gains. This suggests that a pricing scheme based on distance, such as the one used in taxis, is far from e cient. Explicitly targeting origin and destination is essential in order to correct for the di erent sources of externalities.

Related Literature

This paper broadly relates to four strands of literature: search and matching; transportation; international trade; and industry dynamics.

First, our work naturally relates to the search and matching literature; see Diamond (1982), Mortensen (1982) and Pissarides (1985) for the canonical DMP labor market model, as well as Rogerson et al. (2005) for a survey.[1] More speci cally, our paper relates to the literature on e ciency of search models. Hosios matches in every market is optimal. This latter condition is novel. In addition, we derive theoretically the set of policy instruments (both e cient pricing rules, and taxes/subsidies) that can restore e ciency.

Second, our paper contributes to a large and rapidly growing literature on transportation. Our model builds on Lagos (2000) (and Lagos, 2003). More recently, Frechette et al. (2019) and Buchholz (2020) study search frictions and regulation frictions in NYC taxicabs. In particular, Buchholz (2020) relies on a similar framework, and numerically implements tari pricing changes in order to explore whether welfare improvements can be achieved. Frechette et al. (2019) investigate the welfare impact of changes in the number of active medallions, as well as the introduction of an Uber-like platform.

In addition, a series of papers study di erent aspects of e ciency in urban transportation; for instance, Shapiro (2018) and Liu et al. (2019) explore the welfare improvements from di erent centralizing formats; Ghili and Kumar (2020) investigate demand and supply imbalances in ride-sharing platforms; Ostrovsky and Schwarz (2018) focus on carpooling and self-driving cars; Kreindler (2020) studies optimal congestion pricing; Cao et al. (2018) explore competition in bike-sharing platforms; while several papers study platform pricing (e.g. Bian, 2020, Ma et al., 2018, Castillo, 2019).

Third, since our empirical application involves oceanic transportation, we relate to a literature studying transportation in the context of international trade; e.g. Koopmans (1949), Hummels and Skiba (2004), Fajgelbaum and Schaal (2019), Asturias (2018), Brooks et al. (2018), Cosar and Demir (2018), Holmes and Singer (2018), Wong (2018), Allen and Arkolakis (2019), Ducruet et al. (2019), Lee et al. (2020) and BKP. We also relate to a literature in international trade studying the role of frictions, such as Eaton et al. (2016), and Krolikowski and McCallum (2018) who consider search frictions between importers and exporters and Allen (2014) who investigates information frictions. In our prior work, BKP, we explore the role of the transportation sector in world trade and spell out the impact of endogenous trade costs. Although we rely on the model setup and empirical strategy employed there, our focus here is entirely di erent, as this paper considers search frictions and e ciency.

Finally, we relate to the literature on industry dynamics (Hopenhayn, 1992, Ericson and Pakes, 1995), industry dynamics, has explored trading frictions in decentralized markets (e.g. Gavazza, 2011, 2016 for real assets and Brancaccio et al., 2020b for over-the-counter financial markets).

The rest of the paper is structured as follows: Section 2 presents the model. Section 3 provides the efficiency and optimal policy results. Section 4 describes the dry bulk shipping industry and the data used, presents evidence for search frictions and outlines the estimation of the model. Section 5 presents our welfare analysis. Section 6 concludes. The (Online) Appendix contains all proofs and additional theoretical results, evidence on random search in shipping, details on the estimation procedure, data and computation, as well as additional tables and figures.

2 Model

We introduce a model of decentralized transport markets that focuses on the interaction between carriers (e.g. ships, taxis, trucks) and customers (e.g. exporters, passengers).

2.1 Environment

Time is discrete and the horizon is infinite. There are I locations, $i \in \{1, 2, ..., I\}$. There are two types of agents: customers and carriers. Both are risk neutral and have discount factor. Variables with superscript s refer to carriers and e to customers, in line with our empirical exercise of ships and exporters.

There is a measure S of homogeneous carriers in the economy.[4] At the beginning of every period, a carrier is either in some region i, or traveling full or empty, from some location i to some location j. Carriers at i can either search or remain inactive. The per-period payoff of staying inactive is set equal to 0 at each location, while searching carriers incur a per-period search cost c^s. Carriers traveling from i to j incur a per period traveling cost c^s_{ij}. The duration of a trip between location i and location j is stochastic: a traveling carrier arrives at j in the current period with probability d_{ij}, so that the average duration of the trip is $1/d_{ij}$.[5]

[4] A constraint on the fleet size is consistent with most applications of interest, and can be due to either regulatory constraints (e.g. fixed number of medallions) or time to build.

[5] It is straightforward to have deterministic trip durations instead. Our specification, however, preserves tractability and allows for some variability e.g. due to weather/traffic shocks, without affecting the steady state properties of the model.

Customers can only be delivered to their destination by carriers and each carrier can carry at most one customer. Following the search and matching literature, we model the number of matches that take place every period in region i, m_i, using a matching function, whereby

$$m_i = m_i(s_i; e_i) \quad \min\{s_i; e_i\}$$

where s_i is the measure of unmatched carriers in region i and e_i is the number of unmatched customers in region i. $m_i(s_i; e_i)$ is increasing and concave in both arguments. We allow for the possibility that $m_i(s_i; e_i) < \min\{s_i; e_i\}$ creating the potential for unrealized trade: two agents searching in the same location might fail to meet, due to impediments such as information frictions or physical constraints. As Petrongolo and Pissarides (2001) note, [...] the matching function [...] enables the modeling of frictions [...] with a minimum of added complexity. Frictions derive from information imperfections about potential trading partners, heterogeneities, the absence of perfect insurance markets, slow mobility, congestion from large numbers, and other similar factors.

Since search is random, the probability according to which customers searching at i meet a carrier is $^e_i = m_i(s_i; e_i)/e_i$, which is the same for all customers. Similarly the probability according to which carriers searching at i meet a customer is $^s_i = m_i(s_i; e_i)/s_i$.

When a carrier and a customer meet, if they both accept to match, the customer pays a price $_{ij}$ upfront and the carrier begins its trip immediately to j. We are agnostic for now as to what the price mechanism is in the market. This allows us to nest several different practices in different markets; for instance prices are fixed by regulation in taxicabs, while prices are bilaterally negotiated in bulk shipping.

Carriers that remain unmatched decide whether to stay in their current region or travel empty to a different region where they wait for a match. Customers that remain unmatched wait in their current region. Inactive carriers restart the following period in the same region.

Finally, every period, at each location i, a large pool of potential customers decide whether to enter and search for a carrier, in order to be transported to a destination $j \neq i$, subject to an entry cost $_{ij}$. Denote by e_{ij} the endogenous measure of customers in i who search for transportation to j. The total measure of customers searching at location i is $e_i = \sum_{j \neq i} e_{ij}$, while G_{ij} is the share of demand routed from i to j, i.e.,

$$\forall i,j : G_{ij} \geq e_{ij} = e.$$

Once they have entered, customers pay a per-period search cost c^e_{ij}.[6]

Upon matching with a carrier, customers obtain a valuation from being transported from origin i to destination j. We model customer valuations via the function, $w : \mathbb{R}^{I \times I}_+ \to \mathbb{R}^{I \times I}_+$, where $w_{ij}(q)$ is the valuation of the marginal customer on route ij, and q is the matrix with typical element q_{ij} denoting the quantity transported every period (i.e. the measure of accepted matches) on route ij. This can be thought of as an inverse demand curve for transportation services, before customer entry and search costs. For example, consider customers with heterogeneous valuations for transportation (e.g. passengers looking for taxis with different value of time): when q_{ij} matches are formed on route ij, $w_{ij}(q)$ describes the valuation of the q_{ij}-th (i.e. the marginal) consumer entering route ij.[7] As a simpler case, if valuations are homogeneous so that all customers obtain w_{ij} on route ij, the marginal customer naturally also obtains w_{ij}.

Consistent with this interpretation, w is the gradient of a concave and differentiable function $W : q \mapsto \mathbb{R}_+$, which is interpreted as the total customer value from transportation, as a function of the total quantity transported, q.

2.2 Behavior and equilibrium

We consider the steady state of our industry model. In a steady state equilibrium, customers and carriers

Carrier optimality Let V_{ij}^s denote the value of a carrier that begins the period traveling from i to j (empty or loaded), V_i^s the value of a carrier that begins the period in location i, and U_i^s the value of a carrier that remained unmatched at i

steady state is given by $\sum_{ij}(q_{ij}+b_{ij}) = d_{ij}$ (setting $d_{ii}=1$). Hence this condition can be written as,

$$\sum_{ij}\frac{q_{ij}+b_{ij}}{d_{ij}} < S \; ! \; 9 \quad i : V_i^s = 0 : \tag{9}$$

Customer optimality We now turn to the value functions of customers; we begin with existing customers and then consider customer entry. If a customer meets a carrier they can either agree to form a match, in which case the customer pays price $_{ij}$ and receives its valuation, or the customer can revert to its outside option and stay unmatched. Hence the meeting surplus of the marginal customer with valuation $w_{ij}(q)$ is given by,

$${}^{e}_{ij} = \max\left\{ w_{ij}(q) \quad {}_{ij} \quad U^{e}_{ij} ; 0 \right\} ; \tag{10}$$

where U^e_{ij} is its value of searching for a carrier in i with destination j:

$$U^e_{ij} =$$

We adopt the convention that customers in i choosing i do not enter, and normalize the payoff in that case to zero.

Feasible allocations An allocation for the transportation economy consists of a tuple (s, E, q, b) where $s = [s_1, \dots, s_I]$ denotes the measure of carriers waiting in each region, $E \in \mathbb{R}_+^{I \times I}$, with typical element e_{ij}, denotes the measure of customers waiting for transport on each route ij, $q \in \mathbb{R}_+^{I \times I}$ denotes the measure of new matches formed on each route, and $b \in \mathbb{R}_+^{I \times I}$ denotes the measure of carriers departing empty on each route. Equivalently, we will sometimes denote an allocation by (s, e, G, q, b), where $e = [e_1, \dots, e_I] =$ $\left[\sum_j e_{1j}, \dots, \sum_j e_{Ij} \right.$

elsewhere empty $(^{P}\ _{j\,6}$

De nition 2. $(s; E; q; b; \ )$ is a limit equilibrium outcome if there exists a sequence $(s^n; E^n; q^n; b^n; {}^n; {}^n)_{n \ 0}$ such that: (i) for each n, $(s^n; E^n; q^n; b^n; {}^n)$ is an equilibrium outcome for the economy populated by agents with discount factor ^{n}; and (ii) as ${}^n \to 1$, $(s^n; E^n; q^n; b^n; {}^n) \to (s; E; q; b; \ )$. $(s; E; q; b)$ is a limit equilibrium allocation if there exists a price matrix such that $(s; E; q; b; \ )$ is a limit equilibrium outcome.

Theorem 1. If $(s;E;q;b)$ is a limit equilibrium allocation then it solves

$$\max_{s;E;q;b\ 0} W(q) \qquad \sum_{ij} q_{ij}\ {}_{ij} \qquad \sum_{ij}(q_{ij}+b_{ij})\frac{c^s_{ij}}{d_{ij}} \qquad \sum_{i} s_i c^s_i \qquad \sum_{ij} e_{ij}\, c^e_{ij} \tag{20}$$

s.t. feasibility constraints (15)-(17)

$$8i;j\ :\ q_{ij} \qquad {}^s_i s_i G_{ij} \tag{21}$$

$$8i;j\ :\ q_{ij} \qquad {}^e_i e_{ij}\ : \tag{22}$$

where the perceived probabilities ${}^s;\ {}^e$ and G are taken as given and are consistent with the true ones (i.e. they satisfy condition 4 in Definition 1).

Theorem 1 characterizes market equilibrium allocations as solutions to Problem (20), the *market problem*. As in the planner Problem (19), the objective function is equal to total welfare. Moreover, both the market and the planner face the steady state constraints (15)-(16), and the total fleet constraint (17). However, when it comes to the matching constraints, Problems (19) and (20) differ. Indeed, the social planner faces constraint (18), which treats the meeting rates ${}^s;\ {}^e$ and the destination shares G as endogenous objects that are functions of $s;e$; in contrast, constraints (21) and (22) in the market Problem (20) treat these objects as exogenous constants.

The proof of Theorem 1, provided in Appendix A, rests heavily on duality. In particular, the dual variables of the market Problem (20) are linked to the carrier and customer value functions. This, in turn allows us to show that the carrier optimality conditions, equations (1)-(9), and the customer optimality conditions, (10)-(14), are equivalent in the limit to the first order conditions of the market Problem (20).[10]

Importantly, when comparing the market Problem (20), to the planner Problem (19), the only difference is that the latter internalizes the effect of search behavior on the endogenous meeting probabilities and destination shares. The market's failure to optimize with respect to these variables is the unique potential source of inefficiency in the economy.

[10] Caution is needed however when limits are taken as the discount factor goes to one, because the value functions per se may diverge. The desired correction is obtained by subtracting a reference value function from the remaining ones. Detailed arguments are found in the Appendix A.

3.2 Externalities and e cient prices

In contrast to a frictionless world, in an economy with search frictions prices may fail to balance demand

The social planner Problem (19) is equivalent to:[11]

$$\max_{s;e;G \ge 0} V^p(s;e;G)\,; \quad \text{s.t.} \sum_j G_{ij} = 1\ \forall i \text{ and } \sum_i s_i \le S \tag{24}$$

Intuitively, since the only source of inefficiency results from agents' search behavior, it is useful to optimize out the other variables (i.e. $q; b$) in order to focus on the impact of the main variables of interest, $s; e; G$.

Definition 3. At a search allocation $(s; e; G)$:

- Carriers internalize thin/thick market externalities if

$$s \in \arg\max_{s' \ge 0} V^p\left(s', e; G\right) \quad \text{s.t.} \sum_i s_i \le S: \tag{25}$$

- Customers internalize thin/thick market externalities if

$$e \in \arg\max_{e' \ge 0} V^p\left(s; e', G\right): \tag{26}$$

- Customers internalize pooling externalities if

$$G \in \arg\max_{G' \ge 0} V^p\left(s; e; G'\right) \quad \text{s.t.} \sum_j G_{ij} = 1\ \forall i: \tag{27}$$

Our next theorem states three conditions that determine how the meeting surpluses must be shared between carriers and customers in order for the externalities to be internalized in equilibrium. For every $i \in I$, we denote by $\eta_i^s = d\ln m_i(s_i; e_i)/d\ln s_i$ and $\eta_i^e = d\ln m_i(s_i; e_i)/d\ln e_i$, the elasticities of the matching function with respect to the measure of carriers and customers searching at i, respectively. To avoid delving into corner solutions arising in trivial cases, we assume that the equilibrium is such that there is a positive measure of customers and carriers searching at each location ($s_i; e_i > 0\ \forall i$) and that $\sum_i s_i < S$.[12] Let ^{s} and ^{e} denote the carrier and customer limit surpluses associated with the limit equilibrium outcome, (

social surplus. For a formal de nition, see Appendix A.1.

market externalities. Conditions (28) and (29) have a similar avor as the standard Coasian conditions in the presence of externalities, where the private value of an action must be equal to its social value. Indeed, we can rewrite equation (28) as

$$ {}^{s}_{i} \sum_{j} G_{ij} \; {}^{s}_{ij} = dm_i($$

which however creates a wedge between the price paid by the customer and the one received by the carrier.

E cient prices Condition (iv) of Theorem 2 provides a characterization of the e cient pricing rule:

Corollary 1. *Let a limit equilibrium outcome* $(s; e; G; q; b; \ )$ *be e cient. Then we have* $\ _i^s = 1 \quad \ _i^e$ *and*

3.3 Optimal policy under Nash bargaining

In this section we consider the problem of a planner who cannot directly control prices, but can use taxes/subsidies to restore efficiency in the market. We show that the planner can indeed achieve efficiency using such instruments and we derive their optimal values.

Suppose that the planner can impose a tax/subsidy h^q on loaded trips, h^s on searching carriers, and h^e on searching customers. In other words, searching carriers in region i pay h_i^s in addition to their waiting cost c_i^s every period they search; customers searching in i pay h_i^e in addition to their cost c_{ij}^e every period they search; finally, there is a one-time tax h_{ij}^q on every new match (as illustrated below which side pays the tax does not matter).

We focus on a specific price mechanism, that of Nash bargaining, which is a commonly employed model used to capture bilateral negotiations. We can extend the definition of equilibrium to accommodate Nash bargaining and taxes in a straightforward manner: $(s; e; G; q; b; \ )$ is an equilibrium outcome under taxes h and Nash bargaining, if carriers and customers behave optimally given h, $\ $, $\ ^s$, $\ ^e$ and G; the feasibility constraints are satisfied; $\ ^s$, $\ ^e$ and G are consistent with the allocation; and finally, prices are determined by the usual surplus sharing condition,

$$(1 \quad _i) \; _{ij}^s = \; _i \; _{ij}^e \tag{33}$$

where $\ _i$ is the carrier bargaining coefficient at i (see Appendix A.6 for further details).

Corollary 2 derives the tax scheme h that resolves the two externalities:

Corollary 2. Let $(s; e; G; q; b; \ )$ be a limit equilibrium outcome under taxes h and Nash bargaining. Then:

(i) Thin/thick market externalities are internalized if and only if for every i

$$\ _i \sum_j G_{ij} \; _{ij} \left(\frac{h_i^s}{\ _i^s} + \; _i \sum_j G_{ij} \, h_{ij}^q \right) = \; _i^s \sum_j G_{ij} \; _{ij} \tag{34}$$

and similarly,

$$(1 \quad _i) \sum_j G_{ij} \; _{ij} \left(\frac{h_i^e}{\ _i^e} + (1 \quad _i) \sum_j G_{ij} \, h_{ij}^q \right) = \; _i^e \sum_j G_{ij} \; _{ij} : \tag{35}$$

(ii)Pooling externalities are internalized if and only if for all ij

$$h^q_{ij} \quad \sum_j G_{ij}\, h^q_{ij} \quad \frac{i}{\qquad}$$

set the planner revenue in region i, $\sum_j G_{ij} h^q_{ij}$, equal to zero.[17] Multiplying both sides by $(1 \quad _i)$, it is easy to see that Condition (36) requires that the subsidy on route ij that falls on the customer, $(1 \quad _i)(\quad h^q_{ij})$, is equal to the deviation of the carrier surplus, $_i \quad _{ij}$ from the average carrier surplus from i, $_i \sum_j G_{ij} \quad _{ij}$. Therefore, routes where the carrier surplus is high (low) are subsidized (taxed). By setting the customer tax/subsidy equal to the deviation of the carrier surplus, the planner forces the customer to fully internalize the impact of his destination decision on the carrier surplus.

Finally, note that if the planner can only use the search taxes $h^s; h^e$, he can correct the thin/thick market externalities.[18] Similarly if he can tax only matches but not search of any side of the market, then he can correct the pooling externalities (using equation (36) as discussed above). The planner can correct all externalities by taxing matches and either searching carriers or searching customers.[19]

4 Empirical application: dry bulk shipping

In this section we describe our empirical application using data from the dry bulk shipping industry. We begin in Section 4.1 with a description of the industry and the available data. In Section 4.2 we discuss search frictions in this market. In Section 4.3 we brie y discuss model estimation. With the exception of Section 4.2, this section follows closely BKP. Throughout the following sections, unless otherwise noted, we split ports into 15 geographical regions, depicted in Figure 6 of Appendix D.[20]

4.1 Industry description and data

Dry bulk shipping involves vessels designed to carry a homogeneous unpacked dry cargo, for individual shippers on non-scheduled routes. Bulk carriers operate much like taxi cabs: a speci c cargo is transported individually by a speci c ship, for a trip between a single origin and a single destination. Dry bulk shipping

[17] Condition (36) de nes a linear system of equations in terms of the $I \quad 1$ trip taxes h^q_{ij} for each location i. This system has multiple solutions as its rank equals $I \quad 2$. Thus, to obtain a unique solution we would have to impose a linear constraint. Imposing the constraint $\sum_j G_{ij} h^q_{ij} = 0$ is natural as it implies that the budget is balanced in each location.

[18] He can do so by setting $h^e_i = \ ^e_i = (1 \quad _i) \sum_j G_{ij} \quad _{ij} \quad ^e_i \sum_j G_{ij} \quad _{ij}$ and $h^s_i = \ ^s_i + h^e_i = \ ^e_i = (1 \quad ^e_i \quad ^s_i) \sum_j \sum_j G_{ij} \quad _{ij}$:

[19] If he taxes matches and searching carriers, he sets $(1 \quad _i) h^q_{ij} = (1 \quad _i) \quad _{ij} + \sum_j G_{ij} \quad _{ij} \quad _{ij} \quad ^e_i \sum_j G_{ij} \quad _{ij}$, if $G_{ij} > 0$ and $h^s_i = \ ^s_i + \sum_j G_{ij} h^q_{ij} = (1 \quad ^e_i \quad ^s_i) \sum_j G_{ij} \quad _{ij}$:

[20] To determine the regions, we employ a clustering algorithm that minimizes the within-region distance between ports. The regions are: West Coast of North America, East Coast of North America, Central America, West Coast of South America, East Coast of South America, West Africa, Mediterranean, North Europe, South Africa, Middle East, India, Southeast Asia, China, Australia, Japan-Korea. We ignore intra-regional trips and entirely drop these observations.

involves mostly commodities, such as iron ore, steel, coal, bauxite, phosphates, but also grain, sugar, chemicals, lumber and wood chips; it accounts for about half of total seaborne trade in tons (UNCTAD)

Fourth, we use the ERA-Interim archive, from the European Centre for Medium-Range Weather Forecasts (CMWF), to collect global data on daily sea weather. This allows us to construct weekly data on the wind speed (in each direction) on a 6° grid across all oceans.

We provide a brief overview of the data and empirical regularities and we refer the interested reader to BKP for further details. Our final dataset stretches from 2012 to 2016 and involves 5,398 ships (about half the world fleet) and 12,007 shipping contracts with a known price, origin and destination.[23] The average

4.2 Search frictions in dry bulk shipping[24]

A number of features of dry bulk shipping, such as information frictions and port infrastructure, can hinder the matching of ships and exporters. In this section we argue that these frictions indeed lead to unrealized potential trade. Consider a geographical region, such as a country or a set of neighboring countries, where there are s ships available to pick up cargo and e exporters searching for a ship to transport their cargo. We define search frictions by the inequality:

$$m < \min\{s; e\} \tag{38}$$

where m is the number of matched ships and exporters. In other words, under frictions there is potential trade that remains unrealized; in contrast, in a frictionless world, the entire short side of the market gets matched, so that $m = \min\{s; e\}$. When inequality (38) holds, matches are often modeled via a matching function, $m = m(s; e)$, as is done in Section 2 above, and also extensively in the labor literature.

In this section, we present three facts consistent with frictions, as defined by (38). In particular, we (i) provide a direct test for inequality (38); (ii) we document wastefulness in ship loadings; (iii) we document substantial price dispersion. Then, we estimate the matching function $m = m(s; e)$ and gauge the degree of frictions.

Evidence of search frictions We begin with a simple test for search frictions. If we observed all variables $s; e; m$, it would be straightforward to test (38); this is essentially what is done in the labor literature, where the co-existence of unemployed workers and vacant firms is interpreted as evidence of frictions. However in our setup, we observe m (i.e. ships leaving loaded) and s, but not e; we thus need to adopt a different approach.

Assume there are more ships than exporters, i.e. $\min(s; e) = e$. We begin with this assumption, because our sample period is one of low shipping demand and severe ship oversupply due to high ship investment between 2005 and 2008 (see Kalouptsidi, 2014). If there are no search frictions, so that $m = \min(s; e) = e$, small exogenous changes in the number of ships should not affect the number of

[24] The material in this section was included in a previous working version of our paper "Geography, Transportation and Endogenous Trade Costs"; please see NBER Working Paper 23581.

	N	Joint Significance	$\frac{s}{m}$
North America West Coast	193	0	2.706
North America East Coast	200	0	3.172
Central America	199	0.001	3.451
South America West Coast	198	0	2.913
South America East Coast	200	0	4.083
West Africa	200	0.001	5.862
Mediterranean	200	0	4.244
North Europe	200	0	3.577
South Africa	200	0	2.862
Middle East	200	0	3.86
India	200	0.34	8.58
South East Asia	200	0	3.334
China	200	0.038	6.194
Australia	187	0	2.457
Japan-Korea	200	0	5.311

Table 1: Test for search frictions. Regressions of the number of matches in each region on the unpredictable component of weather conditions in the surrounding seas. For each region we use weeks in which there are at least twice as many ships as matches. The first column reports the number of observations; the second column joint significance; and the third column the average ratio of ships over matches in each region during these weeks. To proxy for the unpredictable component of weather, we partition the globe into cells of 9 × 9, and for each cell we collect data on the speed of the horizontal (E/W) and vertical (N/S) component of wind, as well as wave period and height. To control for seasonality, we residualize the weather measurements for each cell on a quarter fixed effect. The potential regressors include one and two weeks lagged values of all the weather measurements for cells in the sea. Finally, we follow Belloni et al. (2012) to select the relevant instruments in each region.

in labor markets, where there is large wage dispersion among workers who are observationally identical. This observation has generated a substantial and influential literature on frictional wage inequality, i.e. wage inequality that is driven by search frictions.[27] Similarly, Table 7 in Appendix D shows that there is substantial price dispersion in shipping contracts. More specifically, at best we can account for about 70% of price variation, controlling for ship size, as well as quarter, origin and destination fixed effects. Moreover, the coefficient of variation of prices within a given quarter, origin and destination triplet is about 30% (23%) on average (median). In the most popular trip, from Australia to China, the weekly coefficient of variation is on average 34% and ranges from 15% to 65% across weeks.

In addition, it is worth noting that the type of product carried affects the price paid and overall more valuable goods lead to higher contracted prices, as shown in the same table. In the absence of frictions, if

[27] See for instance Burdett and Mortensen (1998), Postel-Vinay and Robin (2002), Mortensen (2003) and references therein.

alternative restrictions; see BKP) and that an instrument that shifts the number of ships exists (the weather shocks). The methodology delivers exporters point-wise and the matching function of each location i nonparametrically. We provide a short description of the approach in Appendix C.1 and refer the reader to BKP for further details, as well as Brancaccio et al. (forthcoming) for a guide on the implementation of this approach in this and other settings.[30]

Figure 5 in Appendix C.1 reports our estimates for search frictions. In particular, to measure the extent of search frictions in different regions, we compute the average percentage of weekly unrealized matches; i.e. $(\min\{s_i; e_i\} - m_i)/\min\{s_i; e_i\}$. Search frictions are heterogeneous over space and may be somewhat sizable, with up to 20% of potential matches unrealized weekly in regions like South and Central America and Europe. On average, 13.5% of potential matches are unrealized.[31]

Moreover, we find that the estimated search frictions are positively correlated with the observed within-region price dispersion (0.47), another indicator of search frictions. We also find that frictions are negatively correlated with the Herfindahl-Hirschman Index of charterers (those reported in the Clarksons contract data) in a region (-0.31); this suggests that when the clientele is disperse, frictions are higher. Finally, when we estimate the matching function separately for Capesize (biggest size) and Handysize (smallest size) vessels, we find that for Capesize, where the market is thinner, search frictions are lower.

4.3 Model estimation and results

We make four changes that render the model presented in Section 2 amenable to empirical analysis. First, we impose a specific pricing mechanism, Nash bargaining, with β_i the ship bargaining coefficient in market i. Second, we add randomness to the discrete choice problem for ships of where to ballast, by adding idiosyncratic shocks to equation (4), so that it becomes,

$$U_i^s = \max_j V_{ij}^s + \epsilon_{ij} \tag{39}$$

[30] For an application to labor markets see Lange and Papageorgiou (2020).

[31] It is worth noting that this does not imply that in the absence of search frictions there would be 13.5% more matches, as we would need to take into account the optimal response of ships and exporters. This is simply a measure of the severity of search frictions in different regions.

where $_{ij}$ are drawn i.i.d. from the Type I extreme value distribution with standard deviation . Third, we consider the version of the model with < 1. In Appendix E we demonstrate that our efficiency results hold in this empirical model with discounting and idiosyncratic shocks. Fourth, we also add randomness to the exporters' problem (14), so that they solve the following discrete choice problem of whether and where to export,

$$\max_{j} \left\{ U_{ij}^{e} \quad _{ij} + \quad _{ij} \right\}$$

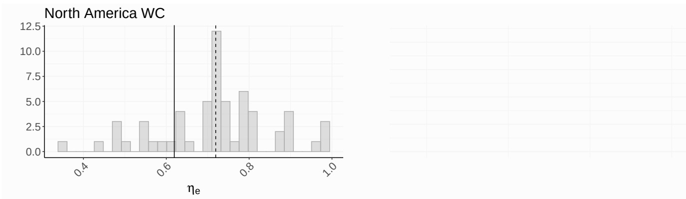
with $_{ij}$ drawn i.i.d. from the Type I extreme value distribution; we normalize $U_{ii}^{e} \quad _{ii} = 0$ and interpret this as the option of not exporting at all. We also assume for simplicity that $w_{ij}(q) = w_{ij}$ for all ij.

The main parameters of interest are: the ship travel and wait costs $c_{ij}^{s}; c_{i}^{s}$, for all $i; j$, as well as the standard deviation of the logit shocks ; the exporter valuations w_{ij}, the exporter waiting costs c_{i}^{e} (to gain power, we assume that c_{ij}^{e} do not vary over j), and entry costs $_{ij}$ for all $i; j$; and the bargaining coefficients $_{i}$ for all i. We present the estimation strategy in Appendix C. Briefly, we use the ship parameter estimates from BKP and estimate the exporter parameters and bargaining coefficients from prices and trade flows. Unlike BKP, we allow the bargaining coefficient to vary by region to allow for flexibility, given the importance of that parameter regarding the thin/thick market externalities. Moreover, we bring in additional data to obtain exporter valuations w_{ij} and as a result we are able to estimate the extra parameters capturing exporter wait costs, c_{i}^{e}.

The results are presented in Table 6 in Appendix D. The exporter wait costs, c_{i}^{e}, are equal to about 3% of the exporters' valuation on average, but there is substantial heterogeneity over space; the estimated costs are highest in Central and South America, as well as parts of Africa. These parameters capture inventory expenditures, delay costs, risks of damage or theft etc. Consistent with this interpretation, we find that exporter wait costs are positively correlated with the recovered wait costs for ships (0.34), and are negatively correlated with the World Bank index of quality of port infrastructure (-0.50). Finally, the estimates for the bargaining coefficients suggest that the exporters get a larger share of the surplus in almost all regions.

5 E ciency in dry bulk shipping

In this section we present our welfare results. In Section 5.1 we check whether the e ciency conditions



	t-stat
North America WC	4.900
North America EC	10.155
Central America	3.002
South America WC	3.497
South America EC	4.080
West Africa	1.169
Mediterranean	6.129
North Europe	7.756
South Africa	1.649
Middle East	6.936
India	8.200
South East Asia	0.685
China	1.290
Australia	1.932
Japan-Korea	2.500

Figure 2: The left panel compares the exporter bargaining coefficient η_i^e and the elasticity of the matching function with respect to exporters, estimated nonparametrically. The histogram corresponds to the estimated elasticity at different points in time. The dotted vertical line is the average elasticity and the solid line is the estimated bargaining coefficient. The right panel presents the t-statistic for the null that the exporter bargaining coefficient η_i^e coincides with the average elasticity of the matching function with respect to exporters.

5.2 Welfare loss

We now come to our main welfare analysis. We begin by a comparison of (i) the market equilibrium; (ii) the constrained efficient outcome we analyzed in Section 3; (iii) the frictionless equilibrium (first-best), i.e., the outcome in a world without search frictions, so that $m = \min\{s; e\}$. To compute the constrained efficient outcome, we compute the equilibrium under the efficient prices given in equation (31) of Corollary

Figure 3: For each region i, we plot the coefficient of variation (standard deviation over mean) of ship surplus for all destinations $j \neq i$. When pooling externalities are internalized, the coefficient of variation should be zero.

1.[32] In terms of policy relevance, one can think of (ii) as what can be achieved by policy makers who are not able to affect the meeting process or the search environment. In contrast, (iii) loosely corresponds to a centralized market; one can think of it as a meeting platform, like Uber, which however does not exercise market power.[33] This three-way comparison allows us to assess both the overall impact of frictions on welfare, as well as the impact of the two externalities under study.

(by 13%), as destinations with high social value are subsidized.

	Frictionless	Constrained E cient	Pooling	Thin/Thick
Welfare	14.32%	6.33%	5.14%	3.29%
Trade	36.50%	13.55%	-13.62%	19.36%
Trade value (net)	42.71%	11.69%	13.61%	6.48%
Ballast miles	-0.60			

the planner subsidizes destinations that are big exporters, implying that the ship can easily reload there, and he taxes destinations that force the ship to ballast afterwards and/or to ballast somewhere far.

example, the planner may not be able to set prices. Moreover, he may be able to tax trips, but not searching agents; indeed, it may be di cult to tax hailing passengers and searching exporters, or waiting taxis/ships. Finally, the matrix h^q may be very large, in which case the planner might prefer a simpler tax scheme.

In this section we consider simple policies that are designed to mimic the optimal taxes, but may be more easily implementable. In particular, we consider the following taxes: (i) an origin-speci c tax on matches which can be interpreted as a at tax on exports; (ii) a destination-speci c tax on matches which can be interpreted as a customs tax; (iii) a linear in distance tax, resembling the taxi price schedule.

Table 3 reports the maximum welfare gains under these tax schemes. The destination-speci c tax works best, as it achieves welfare gains of 2.8%. The origin-speci c tax delivers only 0.9% welfare gains.

naturally to the e cient pricing rules. Moreover, we derive the optimal taxes that restore e ciency for a social planner that cannot set prices. Then, using data from dry bulk shipping, we demonstrate that search frictions are present and lead to a sizeable social loss. However, through optimal taxes/subsidies

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Online Appendix

A Proofs

A.1 Preliminaries: limit equilibrium outcomes and associated dual variables

we have

$$
\begin{aligned}
V_i^{s;n} \quad V_i^{s;n} \quad & c_i^s + {}_i^{s;n} \sum_{j \neq i} G_{ij}^n \; {}_{ij}^{s;n} + \frac{{}^n d_{ii} \;\; V_i^{s;n} \quad c_{ii}^s}{1 \quad {}^n (1 \quad d_{ii} \;\;)} \quad V_i^{s;n} \\
= \quad & c_i^s + {}_i^{s;n} \sum_{j \neq i} G_{ij}^n \; {}_{ij}^{s;n} \quad \frac{c_{ii}^s}{1 \quad {}^n (1 \quad d_{ii} \;\;)} \quad \frac{(1 \quad {}^n) \, V_i^{s;n}}{1 \quad {}^n (1 \quad d_{ii} \;\;)}
\end{aligned}
$$

Note that for every i it holds that

$$\lim_{k\to\infty}(1 \quad {}^{n_k})\,V_i^{s;n_k} = \underbrace{\lim_{k\to\infty}(1 \quad {}^{n_k})\,(V_i^{s;n_k} \quad V_i^{s;n_k})}_{=0} + \lim_{k\to\infty}(1 \quad {}^{n_k})\,V_i^{s;n_k} = \quad :$$

De nition 4. $\quad ; \; ; \; ; \quad {}^{s}; \quad {}^{e}$ is a tuple of equilibrium dual variables associated with the limit equilibrium outcome $(s; E; q; b; \;)$.

Lemma 2.

Subtracting $V_i^{s;n}$ from both sides we obtain,

$$U_i^{s;n} \quad V_i^{s;n} > \frac{{}^n d_{ij} \quad V_j^{s;n} \quad V_i^{s;n}}{1 \quad (1 \quad d_{ij})^{\ \ n}} \quad \frac{c_{ij}^s}{1 \quad (1 \quad d_{ij})^{\ \ n}} \quad \frac{(1 \quad {}^n) V_i^{s;n}}{1 \quad (1 \quad d_{ij})^{\ \ n}}; \quad \text{with equality if } b_{ij}^n > 0$$

Taking limits of both sides as $n ! 1$ yields Condition (41).

As another example, notice that the equilibrium conditions (2), (6) and (7) are equivalent to

$${}_{ij}^{s;n} \quad 0; \quad \text{with equality if } q_{ij}^n < s_i^n \; {}_i^{s;n} G_{ij}^n$$

and

$${}_{ij}^{s;n} \quad {}_{ij}^n + V_{ij}^{s;n} \quad U_i^{s;n}; \quad \text{with equality if } q_{ij}^n > 0:$$

Taking the limit of the rst one gives Condition (42). The second condition can be written as,

$${}_{ij}^{s;n} \quad {}_{ij}^n + V_{ij}^{s;n} \quad V_{i_{s;n}}^{s;n} \atop {}_{is;n}$$

s; E; q; b; ; ; ; ^{s}; ^{e} is an optimal dual pair of Problem (20) (that is, (s; E; q; b) is an optimal solution of Problem (20) and ; ; ; ^{s}; ^{e}

Since $x_1 + (1 \quad) x_2 \in M(u_1) + (1 \quad) M(u_2) \quad M(u_1 + (1 \quad) u_2)$, we have,

$$\inf_{x \in M(u_1 + (1 \quad) u_2)} f(x; u_1 + (1 \quad) u_2) \quad f(x_1 + (1 \quad) x_2; u_1 + (1 \quad) u_2)$$

Since $f(\,)$ is convex in $(x; u)$ we have,

$$\begin{aligned} g(u_1 + (1 \quad) u_2) \quad & f(x_1 + (1 \quad) x_2; u_1 + (1 \quad) u_2) \\ & f(x_1; u_1) + (1 \quad) f(x_2; u_2) \\ & g(u_1) + (1 \quad) g(u_2) + \end{aligned}$$

Since this is true for all , convexity is established. □

Applying this lemma to the function $V^p(s; e; G)$, defined in (23), we obtain that $V^p(s; e; G)$ is concave. Hence, it is differentiable almost everywhere in its domain. Denote by $\partial V^p(s; e; G)$ the supergradient of V^p at a search allocation $s; e; G$, that is, the set of all vectors

$$y = (y(s_i)_{i \in I}; y(e_i)_{i \in I}; y(G_{ij}))_{i;j \in I} \in R^I \quad R^I \quad R^{I \quad I}$$

such that for every search allocation $s^0; e^0; G^0$:

$$V^p \; s^0; e^0; G^0 \quad V^p(s; e; G) \quad \sum_i y(s_i) \; s_i^0 \quad s_i \; + \sum_i y(e_i) \; e_i^0 \quad e_i \; + \sum_{ij} y(G_{ij}) \; G_{ij}^0 \quad G_{ij} \; :$$

Similarly, for every $i; j$, we denote by $\partial_{s_i} V^p(s; e; G)$, $\partial_{e_i} V^p(s; e; G)$ and $\partial_{G_{ij}} V^p(s; e; G)$ the supergradients of V^p at $s; e; G$ with respect to s_i, e_i and G_{ij}, respectively.

Lemma 5. Take a limit equilibrium allocation $(s; e; G; q; b)$, and let $\; ; \; ; \; ; \; ^s; \; ^e$ be a tuple of equilibrium dual variables associated with it. For every $i; j$ define

$$y(s_i) = \quad _i \quad c_i^s + \frac{dm_i(s_i; e_i)}{ds_i} \sum_j G_{ij} \quad _{ij}^s + \; _{ij}^e \; + \; _i$$

$$y(e_i) = \quad c_{ij}^e + \frac{dm_i(s_i; e_i)}{de_i} \sum_j G_{ij} \quad _{ij}^s + \; _{ij}^e$$

$$y(G_{ij}) = \quad e_i c^e_{ij} + m_i(s_i; e_i) \quad {}^s_{ij} + \quad {}^e_{ij} \quad :$$

Then $y \, 2 \, @V^p(s; e; G)$.

Proof. Consider Problem (23) de ning $V^p(s; e; G)$. Its Lagrangian can be written as

$$L \quad q^0; b^0; \; {}^0; \; {}^0; \; {}^0; \; {}^0 j s; e; G \;= W \quad q^0 + \sum_{ij} \quad q^0_{ij} + b^0_{ij} \quad \frac{c^s_{ij}}{d_{ij}} + \quad {}^0_j \quad {}^0_i \quad \frac{}{d_{ij}} \; ! \quad \sum_{ij} \quad q^0_{ij} \quad {}^0_{ij} + \quad {}_{ij}$$

$$\sum_i \quad e_i \sum_j \quad G_{ij} \, c^e_{ij} \quad \sum_i \quad s_i \quad {}^0_i \quad {}^0_i + c^s_i \; + \sum_i \quad m_i(s_i; e_i) \sum_j \quad G_{ij} \quad {}^0_{ij} + S \quad {}^0$$

and the Karush-Kuhn-Tucker (K-K-T) conditions as

$${}_i \quad {}_j \quad \frac{c^s_{ij}}{d_{ij}} \quad \frac{}{d_{ij}} \text{ with equality if } b_{ij} > 0$$

$${}_{ij} \quad 0 \text{ with equality if } q_{ij} < m_i(s_i; e_i) \, G_{ij}$$

$${}_{ij} \quad {}_j \quad \frac{c^s_{ij}}{d_{ij}} \quad {}_{ij} \quad {}_i \quad \frac{}{d_{ij}} \text{ with equality if } q_{ij} > 0$$

$$0 \text{ with equality if } \sum_{ij} \frac{q_{ij} + b_{ij}}{d_{ij}} < S$$

which are equivalent to the set of Conditions (41), (42)/(45), (49) and (44), respectively, taking $\;{}_{ij} = \;{}^s_{ij} + \;{}^e_{ij}$. Since the problem is concave, the K-K-T conditions are necessary and su cient for optimality. Hence letting $\;;\;;\;;\; {}^s;\; {}^e$ be a tuple of equilibrium dual variables associated with $(s; e; G; q; b)$, it follows that $\; q; b; \;;\;; \; {}^s + \; {}^e; \;$ is an optimal dual pair for Problem (23). From the assumptions of Theorem 2 , it follows that $(q; b)$ is the unique optimal solution of Problem (23). Hence the result follows from Theorem 2 of Marimon and Werner (2019). □

We now proceed with the proof of the main result. By the previous analysis, Problem (24) is concave, hence optimality is characterized by the K-K-T conditions. Recall that we are assuming that s and e are in the interior of the feasible set ($s_i; e_i > 0$ for each i and $\sum_i s_i < S$). Hence conditions (25) and (26) are equivalent to the rst order conditions,

$$0 \, 2 \, @_{s_i} V^p(s; e; G) \;\; 8i \;\text{ and }\; 0 \, 2 \, @_{e_i} V^p(s; e; G) \;\; 8i$$

respectively. Denoting by $_{ij}$ and $_i$ the multipliers associated with the constraints G_{ij} 0 and $\sum_j G_{ij} = 1$

A.4 Proof of Corollary 1

Suppose that $(s; e; G; q; b; \;)$ is e cient. Conditions (i) and (ii) of Theorem 2 imply that $\;^{s}_{i} = 1 \;\; ^{e}_{i}$ for all i. For every ij such that $G_{ij} > 0$, Conditions (i) and (iii) of Theorem 2 imply $\;^{s}_{ij} = (1 \;\; ^{e}_{i}) \sum_{j} G_{ij} \;\; _{ij}$. By Condition (48) we have $\;^{e}_{ij} = w_{ij}(q) \;\; _{ij} \;\; _{ij}$. Substituting $\;^{s}_{ij} = \;\; _{ij} \;\; ^{e}_{ij} = \;\; _{ij} \;\; w_{ij}(q) + \; _{ij} + \; _{ij}$ yields Condition (31).

(ii) Customers internalize thin/thick market externalities if and only if

$$\forall i \in I : \frac{\sum_j G_{ij} \; {}^{e}_{ij}}{\sum_j G_{ij} \; {}_{ij}} = {}^{e}_{i}. \tag{55}$$

(iii) Customers internalize pooling externalities if and only if

(54) and (55), and recalling that ${}^{e}_{ij} \quad {}^{s}_{ij} = \quad {}_{ij} \quad {}^{e}_{ij} \quad {}^{s}_{ij}:$

$$\sum_j G_{ij} \quad {}^{e}_{ij} \quad {}^{s}_{ij} = (1 \quad {}^{e}_{i} \quad {}^{s}_{i}) \sum_j G_{ij} \quad {}_{ij}: \qquad (58)$$

This expression can be interpreted as saying that the average price wedge at each location is proportional to the degree of decreasing returns to scale. Under constant returns to scale the wedge is zero: consistently with our main results, e ciency in this case can be achieved by setting a unique price on every route. If the matching function has decreasing returns to scale then the price wedge is positive, imposing a tax on matches at that location, capturing the social cost of making additional matches harder to form because of decreasing returns. On the contrary, matches are subsidized when the matching functions have increasing returns.

Conversely, it is easy to see that equations (57) and (58) imply equations (54)-(56).

We state the conclusions of this section below:

Corollary 3. Let $(s; e; G; q; b; {}^{e}; {}^{s})$

Therefore, the only expressions that change compared to Section 2.2 are the carriers' value of searching:

$$V_i^s = \max \left\{ c_i^s \quad h_i^s + {}_i^s \sum_{j \neq i} G_{ij} \; {}_{ij}^s + U_i^s ; \; V_i^s \right\}$$

and the customers' value of waiting and meeting surplus:

$$U_{ij}^e = \quad c_{ij}^e \quad h_i^e + \; {}_i^e$$

- Thin/thick market externalities are internalized if and only if for all i,

$$\sum_j G_{ij} \ {}^{s}_{ij} = {}^{s}_{i} \sum_j G_{ij} \ {}_{ij} + \frac{h^s_i}{{}^{s}_{i}} \tag{59}$$

and

$$\sum_j G_{ij} \ {}^{e}_{ij} = {}^{e}_{i} \sum_j G_{ij} \ {}_{ij} + \frac{h^e_i}{{}^{e}_{i}}$$

- Pooling externalities are internalized if and only if for all $i; j$,

$$ {}^{s}_{ij} + h^q_{ij} \quad L_i \text{ with equality if } G_{ij} > 0$$

where L_i is an arbitrary constant.

Using the de nition $_{ij} = {}^{e}_{ij} + {}^{s}_{ij} + h^q_{ij}$ and the Nash bargaining condition $(1 \ _i) \ {}^{s}_{ij} = \ _i \ {}^{e}_{ij}$ it follows that $_{ij} = \frac{1}{_i} \ {}^{s}_{ij} + h^q_{ij}$ or ${}^{s}_{ij} = \ _i \ _{ij} \ \ _i h^q_{ij}$. Substituting ${}^{s}_{ij}$ into (59) we obtain the Condition (34). We proceed similarly for customers to obtain Condition (35).

Next, we turn to the relationship ${}^{s}_{ij} + h^q_{ij} \quad L_i$ with equality if $G_{ij} > 0$. LThe constant $^{q}_{ij}$ h_{ij} ij $^{q}_{ij}$(for)nd8.4P. (18 -3.6k

$$_i \; _{ij} + (1 \quad _i)\, h^q_{ij} \qquad _i \sum_j G_{ij} \; _{ij} + (1 \quad _i) \sum_j G_{ij}\, h^q_{ij}$$

which proves (36).

B Random search in bulk shipping

In this section we investigate whether search in bulk shipping is random (or undirected), as assumed in the model of Section 2. We contrast this with the case of directed search (see e.g. Moen, 1997), where carriers choose to search in a speci c market , i.e. a market for customers heading to a speci c destination. Under directed search, pro table markets attract more carriers, thereby reducing their matching probabilities compared to less pro table markets. We can directly test this implication of directed search by checking whether in a given origin, i, ships' waiting time is di erent across destinations j. We use 15 regions, so for a given region there are (up to) 14 possible destinations; therefore there are $\binom{14}{2} = 91$ such equalities to test for every origin i. Using a simple F-test we are only able to reject the null of no di erence for 16% of the equalities.

In addition, we examine the coe cient of variation of matching probabilities within a given origin. Weighted by trade shares, the average coe cient of variation is just 8%. In contrast, the coe cient of variation of trip prices from a given origin is substantially higher and equal to 46%, suggesting that di erences in the attractiveness of di erent types of customers is re ected in prices, but not in matching probabilities, as would be the case in directed search.

C Estimation and computation details

C.1 Model estimation and results

In this section we discuss the estimation of the model. The main parameters of interest are: the matching functions $m_i(s_i; e_i)$ for all i, the ship travel and wait costs $c^s_{ij}; c^s_i$, for all $i; j$, as well as the standard deviation of the logit shocks ; the exporter valuations w_{ij}, the exporter waiting costs c^e_i, and entry costs $_{ij}$ for all $i; j$; and the bargaining coe cients $_i$ for all i. The available data consist of the matches m_i and ships s_i for all i, the ship ballast choice probabilities P_{ij}, for all ij, the average prices $_{ij}$ on all routes ij , the exporter entry probabilities P^e_{ij}, for all ij as well as total trade values by country pair (Comtrade). We describe the estimation of each object in turn.

Matching function estimation We brie y outline the approach adopted to estimate the matching function in BKP. To illustrate, assume that s and e are independent. We assume that $m(s; e)$ is continuous and strictly increasing in e, that it exhibits constant returns to scale (CRS), so that $m(as; ae) = am(s; e)$ for all $a > 0$, and that there is a known point f s; e; mg, such that $m = m(s; e)$. The intuition behind the weather conditions (unpredictable wind at sea) that shift the arrival of ships at a port without affecting the number of exporters (also employed in the search frictions test, see Section 4.2).[39] Table 4 presents the first stage estimates.

Figure 5 reports our estimates for search frictions, along with confidence intervals constructed from 200 bootstrap samples.

	F-stat
North America West Coast	21.132
North America East Coast	18.429
Central America	17.877
South America West Coast	18.671
South America East Coast	16.889
West Africa	16.333
Mediterranean	46.072
North Europe	28.651
South Africa	13.153
Middle East	68.037
India	29.521
South East Asia	34.909
China	28.642
Australia	35.977
Japan-Korea	32.794

Table 4: First Stage, Matching Function Estimation. Regressions of the number of ships in each region on the unpredictable component of weather conditions in the surrounding seas. The table reports the F-statistic. For the construction of the instrument, see Table 1.

Ship parameters We use the estimates for the ship parameters $\left\{c_{ij}^s; c_i^s; \right\}$ from BKP. To estimate these parameters, we used a Nested Fixed Point Algorithm (Rust, 1987): at every guess of the parameters $\left\{c_{ij}^s; c_i^s; \right\}$ for all $i; j$, we employ a fixed point algorithm to solve for the ship value functions $V_i^s; V_{ij}^s; U_i^s$, for all $i; j$ from equations (1), (3), and (39), using the observed average prices for each route and the observed meeting probability $_i^s$ (which is set equal to the average $m_i = s_i$). We then match the ship ballast

[39] Assume that an instrument z exists such that $s = h(z;)$, with z independent of e, . The approach now has two steps. In the first step, we recover using the relationship $s = h(z;)$. In the second step, we repeat the above conditioning on both s (as beoning

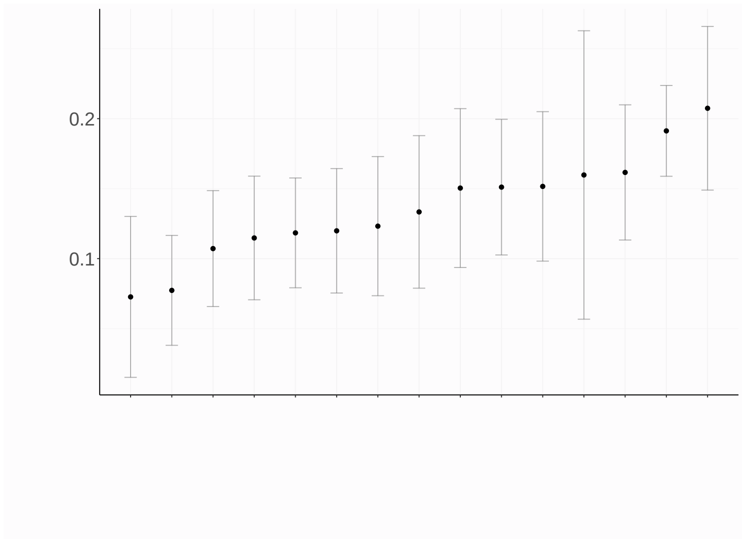

Figure 5: Search Frictions. Average weekly share of unrealized matches, with con dence intervals from 200 bootstrap samples.

choices predicted by our model and given by the logit choice probabilities,

$$P_{ij}^{S} = \frac{\exp\ V_{ij}^{s}=}{\sum_{l} \exp\ V_{il}^{s}=} \qquad (60)$$

to the observed ballast choices. We do so by maximizing over the parameters via Maximum Likelihood.

See BKP for further details on identi cation 4479.22 -39.2o1ec39(iiF24 r79.22.091 Tf 1888 3.757 T290.1J/F]TJ -2.4

and quantities by country pair. We focus on bulk commodities and compute the average value of a cargo of commodities exported from each region i to each j, which forms our direct estimate for w_{ij}; details are provided in the next section.

Next, we turn to c_i^e and

C.2 Exporter valuations

We construct exporter valuations, w_{ij}, from product-level data on export value and quantity by country-pair, obtained from Comtrade. We select bulk commodities among all possible 4-digit HS product codes. The list includes cereals (except rice and barley); oil seeds (which consists of mostly soybeans); cocoa beans; salt and cement; ores; mineral fuels (except petroleum coke); fertilizers; fuel wood and wood pulp; metals; cermets and articles thereof.

To compute the average value of a cargo exported from region i to j, we rst compute the average price of a ton exported by dividing total export value by total export quantity from i to j. Then, we multiply this price by the average ship tonnage capacity in our sample.[42]

Finally, although most countries belong to one of our regions (depicted in Figure 6), the USA and Canada each belong to two regions (according to the coast). We thus need to split the Comtrade data for the USA and Canada into east and west coast export values. To do so, we employ data on state-level exports from the US Census, as well as on province-level exports from the Canadian International Merchandise Trade Database. In particular, we assign every state (province) to either the east or the west coast and compute, for every product, the share of the total value of trade in that commodity that is exported by east and west coast states (provinces). Then, we compute the total value and quantity of trade for the region East Coast of North America (West Coast of North America) by summing over products the share of the value of east (west) coast trade by the total value of the country's trade for the USA and Canada. Implicitly, this approach assumes that export values from these two regions are only di erent due to the composition of products, not their prices.

C.3 Algorithm to compute the e cient allocation

Here, we describe the algorithm employed to compute the steady state of our model. In order to simulate both the market equilibrium and the e cient allocation we approximate the matching function that we obtained non-parametrically with a Cobb Douglas. In particular, for each region we impose

of the relevant commodities for each region i.

[42] This is robust to using the average ship tonnage capacity on route ij.

$m_{it} = A_i s_{it}^{1 \;\; i} e_{it}^{\;\; i}$, and select the parameters $(A_i; \;\; _i)$ through non-linear least-squares using the non-parametrically estimated exporters.

The algorithm proceeds as follows:

1. Make an initial guess for $\{U^{e;0}; \;\; ^0; s^0; E^0\}$.
2. At each iteration k, inherit $\left\{U^{e;k-1}; \;\; ^{k-1}; s^{k-1}; E^{k-1}\right\}$

	Port Costs c_i^s	Sailing Costs c_{ij}^s	Logit Shock	
North America West Coast	227.65	46.75		
	(8.77)	(0.36)		
North America East Coast	272.3	-		
	(4.31)	-		
Central America	175.41	46.75		
	(5.06)	(0.36)		
South America West Coast	265.55	46.75		
	(7.77)	(0.36)		
South America East Coast	292.5	-		
	(5.23)	-		
West Africa	145.3	47.65		
	(4.84)	(0.33)		
Mediterranean	121.89	46.16		
	(3)	(0.28)		
North Europe	122.48	46.16		
	(1.71)	(0.28)		
South Africa	220.11	47.65		
	(7.28)	(0.33)		
Middle East	118.45	46.16		
	(2.14)	(0.28)		
India	97.23	45.93		
	(1.8)	(0.28)		
South East Asia	93.14	40.99		
	(1.02)	(0.28)		
China	91.07	40.89		
	(0.98)	(0.25)		
Australia	193.29	40.99		
	(2.85)	(0.28)		
Japan-Korea	100.41	40.89		
	(1.9ia		93.14	ha3700.36)

	Exporter wait costs c_i^e	Ship bargaining coe cient $_i$	Average exporter value w_i
North America West Coast	83.49 (10.72)	0.384 (0.018)	13,738
North America East Coast	83.49 (10.72)	0.585 (0.012)	12,192
Central America	302.3 (69.28)	0.344 (0.038)	14,350
South America West Coast	302.3 (69.28)	0.259 (0.017)	20,096
South America East Coast	302.3 (69.28)	0.371 (0.042)	6,971
West Africa	396.9	0.292	4,547

D Additional gures and tables

Figure 6: De nition of regions. Each color depicts one of the 15 geographical regions.

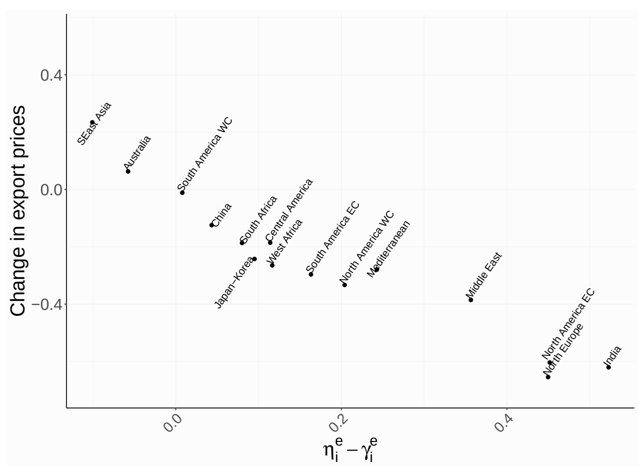

Figure 7: The vertical axis reports the change in prices when only thin/thick market externalities are internalized. The horizontal axis reports the di erence between the estimated exporter bargaining coe cient and the estimated elasticity of the matching function with respect to exporters. We do not allow ships to reallocate to capture the direct e ect of the thin/thick market externalities. See also discussion in footnote 34.

	I	II	III
	log(price per day)		
Probability of ballast		0.234	0.556
		(0.030)	(0.081)
Avg duration of ballast trip (log)		0.166	0.065
		(0.014)	(0.032)
Coal			0.088
			(0.045)
Fertilizer			0.245
			(0.051)
Grain			0.131
			(0.048)
Ore			0.124
			(0.045)
Steel			0.135
			(0.049)
Constant	10.284	9.127	8.915
	(0.103)	(0.099)	(0.408)
Destination FE	Yes	No	No
Origin FE	Yes	Yes	Yes
Ship type FE	Yes	Yes	Yes
Quarter FE	Yes	Yes	Yes
Obs	11,014	11,011	1,662
R^2	0.694	0.674	0.664

** p < 0:05, * p < 0:1

Table 7: Shipping price regressions (Table II in BKP). The dependent variable is the logged price per day in USD.

	log(price per day)			
	I	II	III	IV
$\mathbb{I}\{\text{orig.} = \text{home country}\}$			0.004 (0.019)	
$\mathbb{I}\{\text{dest.} = \text{home country}\}$			0.012 (0.015)	
$\ln(\text{Number Employees})$				0.008 (0.007)
$\ln(\text{Operating Revenues})$				0.003 (0.005)
Time FE	Qtr Yr	Qtr Yr	Qtr Yr	Qtr Yr
Shipowner FE	No	Yes	No	No
Ship characteristics	Yes	Yes	Yes	Yes
Region FE	Orig. & Dest.	Orig. & Dest.	Orig. & Dest.	Orig. & Dest.
Observations	7,263	7,263	7,973	7,973
Adj. R^2	0.530	0.540	0.537	0.537

$p < 0.1$; $p < 0.05$; $p <$

E Supplemental Material: Discounting, preference shocks and out of steady state dynamics

In this section we show that the main results of Section 3.2 are valid in a more general setup. In particular, we extend the model of Section 2 to allow for idiosyncratic preference shocks in carriers relocation choice (relevant in our empirical application), as well as out of steady state dynamics, and we derive an efficiency result analogous to that of Theorem 2.

E.1 Model

We begin by laying out the model focusing on the changes made compared to Section 2.

States and transitions In this Appendix we do not consider the steady state equilibrium. Hence, we now state explicitly the dependence of actions and value functions on the relevant state variables and transitions, which were only implicit in the model of Section 2. At the beginning of a given time period, the state of the economy is described by a vector,

$$z = (x; y) \in \mathbb{R}_+^{I \times I} \times \mathbb{R}_+^{I \times I}:$$

The first element of z, $x = (x_{ij})_{i;j \in I}$, corresponds to the supply at every origin i,

- x_{ii} is the measure of carriers waiting at location i
- x_{ij} is the measure of carriers traveling from i to j, either empty or full, for every destination $j \neq i$.

The second element of z, $y = (y_{ij})_{i;j \in I}$, corresponds to demand. For every origin-destination pair ij, y_{ij} is the measure of customers who are waiting on route ij at the beginning of the current period. These are customers that entered in some previous period and have not yet been matched with a carrier.

At a given state z, the choice sets that agents face, as well as the search and matching process are the

Once a customer and a carrier meet, they can choose whether to match or remain unmatched. The outcome of this process is a vector $(b; q)$ describing the measure of carriers that start traveling empty (b_{ij}) or full (q_{ij}) on each route ij. The state transitions as a function of the allocation $(s; E; q; b)$ are as follows for all ij:

$$x_{ii}^{+1} ($$

to di erent destinations additively, is i.i.d. across carriers and satis es the conditional independence assumption, $_i^{+1} j z^{+1}$? $_i ; z$. To simplify the exposition, we assume that $_i$ is independent of z and i, so that 8$i; z$: $_i$ P 2 R^I, although this assumption is not needed for the results. We assume that P has full support and that it admits a continuous density.

The value of a carrier that remained unmatched at origin i at state z depends on the particular realization of the shock. We denote its expectation by

$$U_i^s ($$

Moreover, they do not reject any match yielding a strictly positive surplus, and they accept only matches yielding a positive surplus:

$$q_{ij}(z) < {}^{s}_{i}(z)\, s_i(z)\, G_{ij} \;!\quad {}^{s}_{ij}(z) = 0$$

$$q_{ij}(z) > 0\,!\quad {}^{s}_{ij}(z) = {}_{ij}(z) + V^{s}_{ij}(z) \quad U^{s}_{i}(z): \tag{67}$$

Customer optimality Customer value functions are the same as in Section 2, but we make the dependence on the state of the economy explicit. In state z, the meeting surplus of the marginal customer (with respect to being unmatched) is given by

$$ {}^{e}_{ij}(z) = \max \overset{n}{\;} w_{ij}(q(z)) \quad {}_{ij}(z) \quad U^{e}_{ij} \;\; z^{+1} \;\; ; 0 \overset{o}{\;} ;$$

where $U^{e}_{ij}(z)$ is the value of customer with destination j that is searching for a carrier in location i:

$$U^{e}_{ij}(z) = \quad c^{e}_{ij} + \; {}^{e}_{i}(z) \;\; {}^{e}_{ij}(z) + \; U^{e}_{ij} \;\; z^{+1} \;\; : \tag{68}$$

Optimality requires that the marginal customer does not reject a match yielding a strictly positive surplus:

$$q_{ij}(z) < {}^{e}_{i}(z)\, {}_{ij}(z) \;!\quad {}^{e}_{ij}(z) = 0: \tag{69}$$

The measure of customers searching on each route ij is pinned down by a free entry condition for the marginal customer:

$$U^{e}_{ij}(z) \quad {}_{ij} \quad 0; \text{ with equality if } e_{ij}(z) > y_{ij}: \tag{70}$$

Equilibrium An outcome is a tuple $(s; E; q; b; \;)$ consisting of an allocation rule and a price rule.

De nition 5. An outcome is a Markovian equilibrium if, for every state z

3. $(E(z); q(z))$ satis es the customer optimality and free entry conditions (68)-(70) given (z); $^{e}(z)$ and z^{+1}.

4. Expectations are consistent with the realized outcomes:

$$8i: \quad {}^{s}_{i}(z) = m_i(s_i(z); e_i(z)) = s_i(z); \quad {}^{e}_{i}(z) = m_i(s_i(z); e_i(z)) = e_i(z)$$

$$z^{+1} = z^{+1}(s(z); E(z); q(z); b(z))$$

that z refers to the sequence of states induced by $s; e; G; q; b$ from z^0:

$$z^{t+1} = z^{+1} \quad s^t; e^t; G^t; q^t; b^t; z^t \quad :$$

for $t \quad 0$. Moreover, when dealing with a feasible allocation rule $(s; e; G; q; b)$ and an initial state z^0, it is understood that $s; e; G; q; b$ refers to the sequence of allocations induced by $(s; e; G; q; b)$ from z^0:

$$s^t; e^t; G^t; q^t; b^t = s \quad z^t \quad ; e \quad z^t \quad ; G \quad z^t \quad ; q \quad z^t \quad ; b \quad z^t$$

be the set of feasible sequences of search allocations, and

$$SA\ z^0 je;G \ = \ {}^{n}\, s : \ s;e;G \ 2\, SA \ z^0 \ {}^{o}$$
$$SA\ z^0 js;G \ = \ {}^{n}\, e : \ s;e;G \ 2\, SA \ z^0 \ {}^{o}$$
$$SA\ z^0 js;e \ = \ {}^{n}\, G : \ s;e;G \ 2\, SA \ z^0 \ {}^{o} \ :$$

For every $s;e;G \ 2\, SA \ z^0$, wl5 Td [n9091 Tf -341
, wl5 Td [n9091 Tf -341
, wl5 Td [n9091 Tf -341
,

to avoid delving into corner conditions, in the statement below we assume that the equilibrium path originating from z^0 is such that we have $s_i^t, e_i^t > 0$ for every t, i.

Theorem 4. Suppose that at a given state z^0, Problem (73) admits a unique optimal solution, and let (s, e, G, q, b) be an equilibrium allocation rule. Then the following statements hold:[47]

(i) Carriers internalize thin/thick market externalities at z^0 if and only if, for every $t \geq 0$:

$$\forall i \in I: \sum_j G_{ij}^t\left(z^t\right) {}^{s}_{ij}\left(z^t\right) = {}^{s}_{i}\left(z^t\right) \sum_j G_{ij}\left(z^t\right)\left[{}^{s}_{ij}\left(z^t\right) + {}^{e}_{ij}\left(z^t\right)\right]:$$

(ii) Customers internalize thin/thick market externalities at z^0 if and only if, for every $t \geq 0$:

$$\forall i \in I: \sum_j G_{ij}\left(z^t\right) {}^{e}_{ij}\left(z^t\right) = {}^{e}_{i}\left(z^t\right) \sum_j G_{ij}\left(z^t\right)\left[{}^{s}_{ij}\left(z^t\right) + {}^{e}_{ij}\left(z^t\right)\right]$$

(iii) Customers internalize pooling externalities at z^0 if and only if, for every $t \geq 0$, for each origin i,

$${}^{s}_{ij}\left(z^t\right) = \max_{k \neq i} {}^{s}_{ik}\left(z^t\right)$$

for every ij such that $G_{ij}\left(z^t\right) > 0$.

The following section provides the proof.

E.3 Proof of Theorem 4

inner product on R^N. De ne the norm k function. Let z^0 2 Z, and $^{N[f \ 0g}$ be such that for every 2 problem

$$P \quad : \max_{x 2 X^{N[f \ 0g}} \sum_{t=0}^{1} {}^{t} u \left(x^t ; {}^{t} ; z^t \right)$$

$$8t; k : f_k \left(x^t ; {}^{t} ; z^t \right) \quad 0$$

$$8t : z^{t+1} = H \left(x^t ; {}^{t} ; z^t \right)$$

is feasible, and letV

of a at T. When dealing with a sequence $;x$ and an initial state z^0, unless stated otherwise, it is understood that z refers to the sequence of states induced by $;x$ and the map H from z^0:

$$\forall t \quad 0 : z^{t+1} = H \quad x^t; \ ^t; z^t \quad :$$

Let $;x;;$ be as in the statement. For every $T > 0$ consider the nite horizon problem,

$$P \quad T; \quad : V^T \quad ^T = \max_{x^T \in X^{T+1}} \sum_{t=0}^{T} \ ^t u \quad x^t; \ ^t; z^t \quad + \quad ^{T+1} \sum_l z_l^{T+1} \ _l^{T+1}$$

$$\text{s.t. } \forall t = 0;:::;T : \forall k : f_k \quad x^t; \ ^t; z^t \qquad 0:$$

By standard convex optimization theory, Conditions (74), (76) and (75) imply that $x^T;\ ^T$ is an optimal dual pair for Problem $P \quad T;$. Hence for every feasible sequence^0 and for every $T > 0$ we have

$$\sum_{t=0}^{T} \ ^t u \quad x^t; \ ^t; z^t \quad + \quad ^{T+1} \sum_l z_l^{T+1} \ _l^{T+1} \qquad \sum_{t=0}^{T} \ ^t u \quad x^{0t}; \ ^t; z^{0t} \quad + \quad ^{T+1} \sum_l z_l^{0T+1} \ _l^{T+1}$$

Sine Z and u are bounded[48], taking limits on both sides implies that x is optimal for P . Hence by our assumptions it must be the unique optimal solution for P . De ne

$$\forall t; n : y_n^t = \frac{\partial u \, x^t; \ ^t; z^t}{\partial_n} + \sum_k \ _k^t \frac{\partial f_k \quad x^t; \ ^t; z^t}{\partial_n} + \sum_l \frac{\partial H_l \quad x^t; \ ^t; z^t}{\partial_n} \ _l^{t+1} :$$

We show that $y \in \partial V$. From Marimon and Werner (2019) it follows that $y^T \in \partial V$ for all $T > 0$:

$$\forall \ ^0 \in \quad : V^T \quad ^0 \qquad V^T \qquad \sum_{t=0}^{T} \ ^t \sum_n y_n^t \quad _n^{0t} \qquad _n^t$$

Pick $^0 \in$ and let x^0 be an optimal solution for $P \quad ^0$. For each T we have

$$\sum_{t=0}^{T} \ ^t u \quad x^{0t}; \ ^t; z^{0t} \quad + \quad ^{T+1} \sum_l z_l^{0T+1} \ _l^{T+1} \qquad V^T \quad ^0$$

[48] u is bounded, being a continuous function on a compact space.

and

$$V^T = \sum_{t=0}^{T} {}^t u\left(x^t; {}^t; z^t\right) + {}^{T+1} \sum_l z_l^{T+1} \; {}_l^{T+1}$$

hence

$$\sum_{t=0}^{T} {}^t u\left(x^{0t}; {}^t; z^{0t}\right) \quad \sum_{t=0}^{T} {}^t u\left(x^t; {}^t; z^t\right) + {}^{T+1} \sum_l \left(z_l^{0T+1} \quad z_l^{T+1}\right) {}_l^{T+1} \quad \sum_{t=0}^{T} {}^t \sum_n y_n^t \left({}^{0t}_n \quad {}^t_n\right)$$

Taking limits of both sides we get $V\ {}^0 \quad V \quad \sum_{t=0}^{1} {}^t \sum_n y_n^t \left({}^{0t}_n \quad {}^t_n\right)$. Since ^{0} was arbitrary, this implies $y \in @V$. Hence, by Lemma 7, maximizes V over if $y = 0$, and this condition is also necessary whenever V is differentiable at . This completes the proof. □

E.3.2 Proof of main result

This subsection is devoted to the proof of Theorem 4. We first establish two auxiliary lemmas.

Lemma 9. The function f defined in equation (71) is continuously differentiable. Moreover, given a vector of choice probabilities $p \in {}^I$, a vector $\in R^I$ and a scalar $\in R$, the following are equivalent:

(i)

$$= E_P \max\left({}_j + {}_j\right) \text{ and } \forall j : p_j = P\left({}_j + {}_j = \max_k\left({}_k + {}_k\right)\right) :$$

(ii)

$$\forall j : f(p) + \frac{@f(p)}{@p_j} \quad \sum_k p_k \frac{@f(p)}{@p_k} + {}_j \quad = 0$$

Proof. It is well known (Galichon, 2018) that

$$\forall p \in {}^I : \quad f(p) = \min \left[\ldots \right.$$

For the second part of the statement, it is known (see Galichon, 2018) that (i) is equivalent to

$$\in \partial(\ f \ (p)) \ \text{ and } \ f \ (p) + \quad = \sum_j p_j \ _j :$$

When f is di erentiable, the condition above is equivalent to (ii). This completes the proof. ☐

Lemma 10. Let $s; e; G; q; b; \ ; \ ^e; \ ^s; \ ; \ ^s; \ ^e$ be such that

$$\lim_{t \to \infty} \ ^t \ ^{s;t}_{ij} = \lim_{t \to \infty} \ ^t \ ^{e;t}_{ij} = 0 \, ;$$

$s; e; G; q; b \in A \quad z^0$ and, for every $t; i; j$, $s_i^t; e_i^t > 0$ and the following conditions hold:

$$^{s;t}_{i} \quad 0 \text{ with equality if } s_i^t < x_{ii}^t$$

$$^{e;t}_{ij} \quad 0 \text{ with equality if } e_i^t G_{ij}^t > y_{ij}^t$$

$$^t_{ij} \quad 0 \text{ with equality if } q_{ij}^t < m_i \ \ s_i^t ; e_i^t \ \ G_{ij}^t$$

$$^t_{ij} \quad ^t w$$

(i) s maximizes the function $s^0 \mapsto V(s^0; e; G; z^0)$ over $SA(z^0|e; G)$ if, for every $i; t$:

$$c^s_i + \frac{\partial m_i(s^t_i; e^t_i)}{\partial s_i} \sum_j G^t_{ij}\ {}^t_{ij} + {}^t_i\ \ {}^{s;t+1}_{ii}\ \ {}^{s;t}_i = 0: \quad (78)$$

This condition is also necessary whenever the function $s^0 \mapsto V(s^0; e; G; z^0)$ is differentiable at s.

(ii) e maximizes the function $e^0 \mapsto V(s; e^0; G; z^0)$ over $SA(z^0|s; G)$ if, for every $i; t$:

$$\frac{\partial m_i(s^t_i; e^t_i)}{\partial e_i} \sum_j G^t_{ij}\ {}^t_{ij} \sum_j G^t_{ij}\ c^e_{ij} + {}_{ij}\ \ {}^{e;t}_{ij}\ \ {}^{e;t+1}_{ij} = 0: \quad (79)$$

This condition is also necessary whenever the function $e^0 \mapsto V(s; e^0; G; z^0)$ is differentiable at e.

(iii) G maximizes the function $G^0 \mapsto V(s; e; G^0; z^0)$ over $SA(z^0|s; e)$ if there exists a sequence t such that, for every $i; t$:

$$m_i(s^t_i; e^t_i)\ {}^t_{ij}\ e^t_i\ c^e_{ij} + {}_{ij}\ \ {}^{e;t}_{ij}\ \ {}^{e;t+1}_{ij}\ \ !^t_i \quad (80)$$

with equality if $G^t_{ij} > 0$:

This condition is also necessary whenever the function $G^0 \mapsto V(s; e; G^0; z^0)$ is differentiable at G.

Proof. We apply Lemma 8 to Problem (73)

$$P(s; e; G) : V^p(s; e; G; z^0) = \max_{q;b} \sum_{t=0}^{\infty} {}^t W^p(s^t; e^t; G^t; q^t; b^t; z^t)$$

$$\text{s.t. } (s; e; G; q; b) \in A(z^0):$$

In doing so, notice that the assumptions of Lemma 8 are satisfied, since by Lemma 9 the function W^p is continuously differentiable, and we can take feasible allocations and states to live inside a compact set.[49]

We use the following notation for the Lagrangian multipliers:

[49] Indeed, let $M = \sum_{ij} x^0_{ij}$. Then for every $(s; e; G; q; b) \in A(z^0)$ we must have

$$\forall t; i; j : 0 \leq s^t_i; q^t_{ij}; b^t_{ij}; x^t_{ij} \leq M:$$

multiplier	constraint
$^{e;t}_{ij}$	$e_i^t G_{ij}^t \quad y_{ij}^t$
$^{s;t}_{i}$	$x_{ii}^t \quad s_i^t$
$^{t}_{i}$	$s_i^t = \mathrm{P}_{j} \; q_{ij}^t + b_{ij}^t$
$^{t}_{ij}$	$q_{ij}^t \quad m_{\substack{i\\i}}$

and the set of Conditions (75) is given by

$$\forall t; i; j \;:\; {}^{t}_{ij} \quad w_{ij} \quad q^{t} \;+\; {}^{s;t}_{ij} \quad {}^{e;t+1}_{ij} \quad {}^{t}_{i} \text{ with equality if } q^{t}_{ij} > 0$$

$$f \quad P^{b}_{i} \;+\; \frac{\partial f \;\; P^{b}_{i}}{\partial P_{ij}} \quad \sum_{k} P^{b}_{ik} \frac{\partial f \;\; P^{b}_{i}}{\partial P_{ik}} \;+\; {}^{s;t}_{ij} \quad {}^{t}_{i} = 0 :$$

Proof of main result In order to prove the main result, let everything be as in the statement. Let $\{V^{s;t}; U^{e;t}; \ ^{s;t}; \ ^{e;t}\}_{t=0}^{\infty}$ be the sequence of carriers and customers' value functions and meeting surpluses associated with the sequence $s; e; G; q; b$ evaluated at the state trajectory z^t, $t >= 0$. For every $t \geq 0$ define $\ ^{s;t} = V^{s;t}$, $\ ^{e;t} = U^{e;t}$, $\ ^{t} = E_P U^{s;t}(\ )$, $\ ^{t} = \ ^{s;t} + \ ^{e;t}$ and

$$
\ _i^{s;t} = \max \left\{ c_i^s + \ _i^s \ z^t \ \sum_{j \neq i} G_{ij}^t \ _{ij}^{s;t} + U_i^{s;t} \quad V_i^{s;t+1}; 0 \right\}
$$

$$
\ _{ij}^{e;t} = \ _{ij} \quad U_{ij}^{e;t}.
$$

Then $s; e; G; q; b; \ ; \ ^e; \ ^s; \ ; \ ^s; \ ^e$ satisfies the conditions of Lemma 10. Moreover, notice that:

- Condition (78) can be written as

$$
\forall i; t: \frac{\partial m_i \ s_i^t; e_i^t}{\partial s_i} \sum_j G_{ij}^t \quad \ _{ij}^{s;t} + \ _{ij}^{e;t} \qquad \ _i^s \ z^t \ \sum_{j \neq i} G_{ij}^t \ _{ij}^{s;t} = 0.
$$

Using $\ _i^s \ z^t = m_i \ s_i^t; e_i^t \ = s_i^t$ and rearranging, this is equivalent to

$$
\ _i^s \ z^t \ \sum_j G_{ij}^t \quad \ _{ij}^{s;t} + \ _{ij}^{e;t} = \sum_j G_{ij}^t \ _{ij}^{s;t}.
$$

- Condition (79) can be written as

$$
\forall i; t: \frac{\partial m_i \ s_i^t; e_i^t}{\partial e_i} \sum_j G_{ij}^t \quad \ _{ij}^{s;t} + \ _{ij}^{e;t} \qquad \sum_j G_{ij}^t \quad c_{ij}^e + U_{ij}^{e;t} \quad U_{ij}^{e;t+1} \quad = 0.
$$

Using $U_{ij}^{e;t} = \ c_{ij}^e + \ _i^e \ z^t \ \ _{ij}^{e;t} + \ U_{ij}^{e;t+1}$, $\ _i^e \ z^t = m_i \ s_i^t; e_i^t \ = e_i^t$ and rearranging, this is equivalent to

$$
\ _i^e \ z^t \ \sum_j G_{ij}^t \quad \ _{ij}^{s;t} + \ _{ij}^{e;t} = \sum_j G_{ij}^t \ _{ij}^{e;t}.
$$

- Condition (80) can be written as

$$
\forall i; j; t: m_i \ s_i^t; e_i^t \quad \ _{ij}^t \quad e_i^t \ c_{ij}^e + U_{ij}^{e;t} \quad U_{ij}^{e;t+1} \qquad ! \ _i^t
$$

$$
\text{with equality if } G_{ij}^t > 0.
$$

Using $U_{ij}^{e;t} = \; c_{ij}^{e} + \; {}_{i}^{e} \; z^{t} \quad {}_{ij}^{e;t} + \; U_{ij}^{e;t+1}, \quad {}_{i}^{e} \; z^{t} \; = \frac{m_i(s_i^t; e_i^t)}{e_i}$ and rearranging, this is equivalent to

$$8i; j; t \; : \; {}_{ij}^{s;t} \qquad \frac{! \, {}_{i}^{t}}{{}_{i}^{e}(z^{t})}:$$

with equality if $G_{ij}^{t} > 0$:

This completes the proof.