Mechanism Design meets Priority Design: Redesigning the US Army's Branching Process*

Kyle Greenberg　　Parag A. Pathak　　Tayfun Sönmez†

June 2021

Abstract

Army cadets obtain occupations through a centralized process. Three objectives – increasing retention, aligning talent, and enhancing trust – have guided reforms to this process since 2006. West Point's mechanism for the Class of 2020 exacerbated challenges implementing Army policy aims. We formulate these desiderata as axioms and study their implications theoretically and with administrative data. We show that the Army's objectives not only determine an allocation mechanism, but also a speci c priority policy, a uniqueness result that integrates mechanism and priority design. These results led to a re-design of the mechanism, now adopted at both West Point and ROTC.

*All opinions expressed in this manuscript are those of the authors and do not represent the opinions of the United States Military Academy (USMA), United States Cadet Command, the United States Army, or the Department of Defense. We are grateful for excellent research assistance from Kate Bradley and Robert Upton. Eryn Heying provided superb help with research administration. The Army's Of ce of Economic and Manpower Analysis provided administrative branching data for this project to Kyle Greenberg as part of a restricted use agreement with USMA and MIT that speci es that data can only be stored, accessed, and analyzed within USMA's information system. Any parties interested in accessing this data must make a direct application to USMA. We are grateful to Scott Kominers for helpful conversations. Pathak acknowledges support from the National Science Foundation for this project.

†Greenberg: Department of Social Sciences, United States Military Academy, email: kyle.greenberg@westpoint.edu. Pathak: Department of Economics, MIT and NBER, email: ppathak@mit.edu, Sönmez: Department of Economics, Boston College, email: sonmezt@bc.edu.

1 Introduction

Each year, the US Army assigns thousands of graduating cadets from the United States Military Academy (USMA) at West Point and the Reserve Officer Training Corps (ROTC) to their first job in a military occupation, or branch, through centralized systems. Combined, the West Point and ROTC branching systems determine the branch placements for 70 percent of newly commissioned Army officers (DoD, 2020). In 2006, the US Army created a "market-based" system for branch assignments with the goal of increasing officer retention (Colarruso, Lyle, and Wardynski, 2010). The system, known as the Branch-of-Choice or BRADSO program, gives cadets heightened priority for a fraction of a branch's positions if they express a willingness to BRADSO, or extend the length of their service commitment.[1]

Since the allocation problem involves both branch assignment and length of service commitment, the Army's branching system is a natural application of the matching with contracts framework developed by Kelso and Crawford (1982) and Hatfield and Milgrom (2005). In that framework, a centralized mechanism assigns both positions and contractual terms. However, the Army's mechanism, hereafter USMA-2006, was designed while the matching with contracts model was still being developed and the original formulation in Hatfield and Milgrom (2005) did not directly apply to the Army's problem. Subsequent research by Hatfield and Kojima (2010) broadened the framework in a way that allows it to apply to the Army's problem.[2] Building on this research, Sönmez and Switzer (2013) proposed that the Army use the cumulative offer mechanism to assign cadets to branches. While this proposal had desirable theoretical properties, it required a more complex strategy space in which cadets have to rank branches and terms jointly. Under the USMA-2006 mechanism, cadets only rank branches and separately indicate their willingness to BRADSO for any branch. The Army considered the existing strategy space more manageable than a more complex alternative. In addition, Sönmez and Switzer (2013) showed that the Nash equilibrium outcome of the USMA-2006 mechanism was equivalent to the outcome of the cumulative offer mechanism if cadet preferences took a particular form, where willingness to BRADSO is secondary to rankings of branches. Seeing the proximity between USMA-2006 and the proposal, the Army decided to keep the simpler strategy space and maintain the USMA-2006 mechanism.

In 2012, the US Army introduced Talent-Based Branching to develop a "talent market" where additional information about each cadet influences the priority a cadet receives at a branch (Colarusso, Heckel, Lyle, and Skimmyhorn, 2016). In the branch assignment process, prioritization at each branch has long been based on the order-of-merit list (OML), a composite of a cadet's academic, physical, and military performance scores. Talent-Based Branching was introduced to allow branches and cadets to better align their interests and fit for one another. Under Talent-ratings of cadets were originally a pilot initiative, but for the Class of 2020, the US Army decided to use these ratings to adjust the underlying OML-based prioritization, constructing priorities at each branch first by the tier and then by the OML within the tier.

The desire to use branching to improve talent alignment created a new objective for the Branch-of-Choice program beyond retention. Since the decision to integrate cadet ratings into the mechanism took place under an abbreviated timeline, the US Army maintained the same strategy space for the mechanism as in previous years, and devised the USMA-2020 mechanism to accommodate heterogenous branch priorities. In their design, the Army created two less-than-ideal theoretical possibilities in the USMA-2020 mechanism. First, a cadet could be charged BRADSO under the USMA-2020 mechanism even if she does not need heightened priority to receive a position at that branch. While this was also possible under USMA-2006, it was nearly four times as common under USMA-2020. Second, under USMA-2020, a cadet's willingness to BRADSO for a branch can improve priorities even for regular positions. Surveys of cadets showed that these aspects potentially undermined trust in the branching system, and led the Army to reconsider the cumulative offer mechanism, despite its more complex strategy space. At that point, the Army established a partnership with market designers.

This paper reports on the design of a new branching system for the Class of 2021, COM-BRADSO, based on the cumulative offer mechanism together with a choice rule for each branch that reflects the Army's dual objectives of retention and talent alignment. We develop a model that integrates priority design with mechanism design. Our main formal result is that the Army's objectives, when formulated through intuitive axioms, uniquely give us the cumulative offer mechanism together with a choice rule, endogenous in our setting. In developing this result, we provide direct evidence of the relevance of these axioms in the design. To the best of our knowledge, our main result is the first joint characterization of the cumulative offer mechanism along with a specific choice rule that is induced by the central planner's policy objectives.[3]

A second contribution of this paper is to provide a formal analysis of the USMA-2020 mechanism. Our analysis shows how issues related to the lack of incentive compatibility became more pressing with the USMA-2020 mechanism, leading the Army to abandon this mechanism. We il-

contained in Appendix A.

2 Model

There is a finite set of cadets I and a finite set of branches B. There are q_b identical positions at any given branch $b \in B$, and a total of $\sum_{b \in B} q_b$ positions across all branches. Each cadet is in need of at most one position, and she can be assigned one at any branch either at a base cost of t^0 years of mandatory servicese

1. for any $i, j \in I$ and $t \in T$,

$$(i,t) \; \omega_b^+ \; (j,t) \quad \Longleftrightarrow \quad i \; \pi_b \; j \quad \text{and}$$

2. for any $i \in I$,

$$(i,t^+) \; \omega_b^+ \; (i,t^0).$$

Let Ω_b^+ be the set of all linear orders on $I \times T$ which satisfy these two conditions.

When a given BRADSO policy is invoked at a branch $b \in B$ (for some or all of the positions), (i) the relative priority order of cadets with identical willingness to serve the increased cost remain the same as the baseline priority order π_b, and (ii) any cadet has higher claims for a position at branch b with the increased cost t^+ compared to her claims for the same position with the base cost t^0.

How much of an advantage a BRADSO policy grants to a cadet in securing a position at branch b due to her willingness to serve the increased cost t^+ differs between distinct elements of Ω_b^+. Given two BRADSO policies $\omega_b^+, \nu_b^+ \in \Omega_b^+$, the policy ν_b^+ has weakly more effective BRADSO than the policy ω_b^+ if,

$$\text{for any } i, j \in I, \qquad (i,t^+) \; \omega_b^+ \; (j,t^0) \implies (i,t^+) \; \nu_b^+ \; (j,t^0).$$

That is, the boost received under ν_b^+ (for the units the BRADSO policy is invoked) is at least as much as the boost received under ω_b^+ for any individual when ν_b^+ has weakly more effective BRADSO than ω_b^+.

2.3 Examples of BRADSO Policies: Ultimate and Tiered

Given a branch $b \in B$ and a baseline priority order $\pi_b \in \Pi$, define the ultimate BRADSO policy $\overline{\omega}_b^+ \in \Omega_b^+$ as the BRADSO policy where willingness to serve the increased cost t^+ overrides any differences in cadet ranking under branch-b baseline priority order π beder6372 0 T74ividualr63 721(135 10.p

$I_b^1, I_b^2, \ldots, I_b^n$ so that, for any two tiers $\ell, m \in \{1, \ldots, n\}$ and pair of cadets $i, j \in I$,

$$\left.\begin{array}{l} \ell < m, \\ i \in I_b^\ell, \text{ and} \\ j \in I_b^m \end{array}\right\} \implies i \; \pi_b \; j.$$

Under a tiered BRADSO policy ω_b^+, for any tier $\ell \in \{1, \ldots, n\}$ and three cadets $i, j, k \in I$,

$$\left.\begin{array}{l} i \; \pi_b \; k, \\ j \; \pi_b \; k, \text{ and} \\ i, j \in I_b^\ell \end{array}\right\} \implies \Big((k, t^+) \; \omega_b^+ \; (i, t^0) \iff (k, t^+) \; \omega_b^+ \; (j, t^0) \Big)$$

bilateral match between cadet i and branch b at the cost of t. Let

$$\mathcal{X} = I \times B \times T$$

denote the set of all contracts. Given a contract $x \in \mathcal{X}$, let $i(x)$ denote the cadet, $b(x)$ denote the branch, and $t(x)$ denote the cost of the contract x. That is, $x = \big(i(x), b(x), t(x)\big)$.

For any cadet $i \in I$, let

$$\mathcal{X}_i = \{x \in \mathcal{X} : i(x) = i\}$$

denote the set of contracts that involve cadet i. Similarly, for any branch $b \in B$, let

$$\mathcal{X}_b = \{x \in \mathcal{X} : b(x) = b\}$$

denote the set of contracts that involve branch b. Observe that for any cadet $i \in I$, her preferences $\succ_i \in Q$ originally defined over $B \times T \cup \{\emptyset\}$ can be redefined over $\mathcal{X}_i \cup \{\emptyset\}$ (i.e. her contracts and remaining unmatched) by simply interpreting a branch-cost pair $(b, t) \in B \times T$ in the original domain as a contract between cadet i and branch b at cost t in the new domain.

2.5 Allocations, Mechanisms, and their Desiderata

An *allocation* is a (possibly empty) set of contracts $X \subseteq \mathcal{X}$, such that

(1) for any $i \in I$, $|\{x \in X : i(x) = i\}| \leq 1$,

(2) for any $b \in$ b(x ||) = i}| 1,

(2) for any b 2b(x TJ/F117 10 0 Td [(b)]TJ/F149 11.3673 Tf 5.773 0 Td [(()]TJ/F131 10.9091 Tf 4.865

A *mechanism* is a strategy space S_i for each cadet $i \in I$ along with an outcome function

$$\varphi : \prod_{i \in I} S_i \to \mathcal{A}$$

that selects an allocation for each strategy profile. Let $S = \prod_{i \in I} S_i$.

Given a mechanism (S, φ), the resulting assignment function $\varphi_i : S \to (B \times T) \cup \{\emptyset\}$ for cadet $i \in I$ is defined as follows: For any $s \in S$ and $X = \varphi(s)$,

$$\varphi_i(s) = X_i.$$

A *direct mechanism* is a mechanism where $S_i = \mathcal{Q}$ for each cadet $i \in I$.

We next formulate the desiderata for allocations and mechanisms. Our first three axioms are basic, and standard in the literature.

Definition 1. An allocation $X \in \mathcal{A}$ satisfies *individual rationality* if, for any $i \in I$,

$$X_i \succeq_i \emptyset.$$

A mechanism (S, φ) satisfies *individual rationality* if, the allocation $\varphi(s)$ satisfies individual rationality for any strategy profile $s \in S$.

Definition 2. An allocation $X \in \mathcal{A}$ satisfies satisfies *non-wastefulness* if for any $b \in B$ and $i \in I$,

$$\left.\begin{array}{c} |\{x \in X : b(x) = b\}| < q_b, \text{ and} \\ X_i = \emptyset \end{array}\right\} \implies \emptyset \succeq_i (b, t^0).$$

A mechanism (S, φ) satisfies *non-wastefulness* if, the allocation $\varphi(s)$ satisfies non-wastefulness for any strategy profile $s \in S$.

Definition 3. An allocation $X \in \mathcal{A}$ has *no priority reversals* if, for any $i, j \in I$, and $b \in B$

$$\left.\begin{array}{c} b(X_j) = b, \text{ and} \\ X_j \succeq_i X_i \end{array}\right\} \implies j \,\pi_b\, i.$$

A mechanism (S, φ) has *no priority reversals* if, the allocation $\varphi(s)$ satisfies elimination of priority reversals for any strategy profile $s \in S$.

This condition states that if cadet j is assigned branch b at any cost and cadet i prefers cadet j's assignment to her own, then j must have higher baseline priority than i.[9] If instead cadet i strictly prefers cadet j's assignment even though cadet j

branch $b \in B$, cadets who are willing to extend their Active Duty Service Obligation (ADSO) by three years if assigned to branch b are given higher priority.[10] To infer which cadets are willing to serve the additional three years of ADSO for any given branch b, the strategy space of the

IC) if, for any $s = \left(P_j, B_j\right)_{j \in I} \in (\mathcal{P} \times 2^B)^{|I|}$, $i \in I$, and $b \in B$,

$$\varphi_i(s) = (b, t^+) \implies \varphi_i\left((P_i, B_i \setminus \{b\}), s_{-i}\right) \neq (b, t^0).$$

That is, any cadet $i \in I$ who receives a position at branch b at the increased cost t^+ under φ should not be able to profit by receiving a position at the same branch at the cheaper base cost t^0 by dropping branch b from the set of branches B_i for which she has indicated willingness to serve the increased cost t^+. Alternatively, a cadet should never be charged BRADSO for a branch merely because of his/her willingness to serve the increased cost.

Our next axiom formulates the idea that the willingness to serve the increased cost t^+ at a branch should never serve the sole purpose of enabling an assignment in this branch at the base cost t^0.

Definition 7. A quasi-direct mechanism φ satisfies *elimination of strategic BRADSO* if, for any $s = \left(P_j, B_j\right)_{j \in I} \in (\mathcal{P} \times 2^B)^{|I|}$, $i \in I$, and $b \in B$,

$$\varphi_i(s) = (b, t^0) \implies \varphi_i\left((P_i, B_i \setminus \{b\}), s_{-i}\right) = (b, t^0).$$

That is, any cadet $i \in I$ who receives a position at branch b at the base cost t^0 under φ should still do so upon dropping branch b from the set of branches B_i for which she has indicated willingness to serve the increased cost t^+ (in case $b \in B_i$).[11] Whenever this axiom fails for a cadet $i \in I$ at a branch $b \in B$, cadet i has an opportunity to strategically indicate a willingness to serve the increased cost t^+ at branch b and receive a position at this branch at the base cost t^0 which is otherwise beyond reach in the absence of this strategy.

Our last axiom relaxes the lack of priority reversals formulated in Section 2.5 by removing any dependence on cadet preference information on branch-cost pairs not solicited by the mechanism.

Definition 8. A quasi-direct mechanism φ has *no detectable priority reversals* if, for any $s = \left(P_j, B_j\right)_{j \in I} \in (\mathcal{P} \times 2^B)^{|I|}$, $b \in B$, and $i, j \in I$,

$$\left.\begin{array}{c} \varphi_j(s) = (b, t^0), \text{ and} \\ \varphi_i(s) = (b, t^+) \quad \text{or} \quad b \; P_i \; b\left(\varphi_i(s)\right) \end{array}\right\} \implies j \; \pi_b \; i.$$

This condition requires that whenever a cadet $j \in I$ is assigned a position at a branch $b \in B$ at the cheaper base cost t^0, while another cadet $i \in I$ receives a visibly less desired assignment by

(i) either receiving a position at the same branch at the increased cost t^+ or

(ii) by receiving a position at a strictly less preferred (and possibly empty) branch based on cadet i's submitted preferences P_i on $B \cup \{\emptyset\}$,

cadet j must have higher baseline priority under branch b than cadet i.

[11] This statement holds vacuously if $b \notin B_i$.

The distinction between our axiom on the lack of priority reversals and its weaker version on the lack of detectable priority reversals is subtle. When a mechanism has priority reversals, thus failing the stronger of the two axioms, there is a cadet $i \in I$ who strictly prefers the assignment of another cadet $j \in I \setminus \{i\}$ despite having higher claims for this position. The key difference is that verification of this anomaly may require knowing the preferences $\succ_i \in \mathcal{Q}$ of cadet i over branch-cost pairs, which is potentially private information that may not be always available (even to the central planner). Verification is particularly challenging if the mechanism is not a direct mechanism. In contrast, when a quasi-direct mechanism has detectable priority reversals, thus failing the weaker of the two axioms, there is a cadet $i \in I$ who strictly prefers the assignment of another cadet $j \in I \setminus \{i\}$ no matter what cadet i's preferences $\succ_i \in \mathcal{Q}$ over branch-cost pairs are provided that they are consistent with her submitted preferences $P_i \in \mathcal{P}$ over branches alone. In that sense, all detectable priority reversals can be verified under a quasi-direct mechanism, but the same is not true for all priority reversals.

3.2 USMA-2006 Mechanism

We are ready to introduce the quasi-direct mechanism the Army has adopted at USMA starting with the Class of 2006 to implement its BRADSO program. Since it is a quasi-direct mechanism, the strategy space for this mechanism is given as

$$S^{2006} = \left(\mathcal{P} \times 2^{\mathcal{B}}\right)^{|I|},$$

and the following construction is useful to introduce its outcome function:

Given an OML π and a strategy profile $s = (P_i, B_i)_{i \in I} \in S^{2006}$, for any branch $b \in \mathcal{B}$ construct the following adjusted priority order $\pi_b^+ \in \Pi$ on the set of cadets I. For any pair of cadets $i, j \in I$,

1. $b \in B_i$ and $b \in B_j \implies i \; \pi_b^+ \; j \iff i \; \pi \; j$,
2. $b \notin B_i$ and $b \notin B_j \implies i \; \pi_b^+ \; j \iff i \; \pi \; j$, and
3. $b \in B_i$ and $b \notin B_j \implies i \; \pi_b^+ \; j$.

Under the adjusted priority order π_b^+, any pair of cadets are rank ordered through the OML π if they have indicated the same willingness to serve for branch b, and through the ultimate BRADSO policy $\overline{\omega}_b^+$ (which gives higher priority to the cadet who has indicated to serve the increases cost) otherwise.

Given an OML π and a strategy profile $s = (P_i, B_i)_{i \in I} \in S^{2006}$, the outcome $\varphi^{2006}(s)$ of the USMA-2006 mechanism is obtained with the following sequential procedure:

Branch assignment: At any step $\ell \geq 1$ of the procedure, the highest π-priority cadet i who is not tentatively on hold for a position at any branch applies to her

1. While in theory the USMA-2006 mechanism has BRADSO-IC failures and detectable priority reversals, these issues have been relatively rare in practice. For example, each year on average 22 cadets have been affected by BRADSO-IC failures and 20 cadets have been affected by detectable priority reversals under the USMA-2006 mechanism across the Classes of 2014-2019 (These facts are described in further detail below in Figure 1).

2. Any potential BRADSO-IC failure or detectable priority reversal can be manually corrected ex-post, since each only involves a cadet needlessly paying the increased cost at her assigned branch. An ex-post manual reduction of the cost to the base cost t^0 completely resolves the issue.

3. Even though the USMA-2006 mechanism allows for additional priority reversals which may alter a cadet's branch assignment and consequently cannot be manually corrected ex-post, the veri cation of any such theoretical failure relies on cadet preferences over branch-cost pairs. Since USMA-2006 is a quasi-direct mechanism, information on cadet preferences over branch-cost pairs is not available.

In summary, any possible failure of the properties above under the USMA-2006 mechanism can either be manually corrected ex-post or cannot be veri ed based on the existing data. In large part for these reasons, the USMA-2006 mechanism was maintained by the Army for fourteen years until the Class of 2020efer5eoalCoB202B38(239()2d6o)-33aeonyear on

A key distinction between the USMA-2006 mechanism and the USMA-2020 mechanism was that, even though the Army continued to cap the number of BRADSO-eligible positions at 25 percent of the total number of positions within each branch, the Army used the adjusted priority ranking of cadets mainly intended for the BRADSO-eligible positions also for the regular positions. Through this practice the matching aspect of the branching process was transformed into a standard priority-based assignment problem, which in turn made it possible for the Army to use the individual-proposing deferred acceptance algorithm to determine the branch assignments.

For any strategy profile $s = (P_i, B_i)_{i \in I}$, the outcome $\varphi^{2020}(s)$ of the USMA-2020 mechanism is given as follows. For any cadet $i \in I$,

$$\varphi_i^{2020}(s) = \begin{cases} \emptyset & \text{if } \mu(i) = \emptyset, \\ (\mu(i), t^0) & \text{if } \mu(i) \notin B_i \text{ or } |\{j \in I : \mu(j) = \mu(i),\ \mu(j) \in B_j, \text{ and } i\ \pi_{\mu(i)}\ j\}| \geq q^+_{\mu(i)}, \\ (\mu(i), t^+) & \text{if } \mu(i) \in B_i \text{ and } |\{j \in I : \mu(j) = \mu(i),\ \mu(j) \in B_j, \text{ and } i\ \pi_{\mu(i)}\ j\}| < q^+_{\mu(i)}. \end{cases}$$

In the USMA-2020 mechanism, each cadet $i \in I$ is asked to submit a preference relation $P_i \in \mathcal{P}$ along with a (possibly empty) set of branches $B_i \in 2^B$ for which she indicates her willing to serve the increased cost t^+ to receive preferential admission. A priority order π_b^+ of cadets is constructed for each branch b by adjusting the baseline priority order π_b using the BRADSO policy ω_b^+ whenever a pair of cadets submitted different willingness to serve the increased cost t^+ at branch b. Cadets' branch assignments are determined by the individual-proposing deferred acceptance algorithm using the submitted profile of cadet preferences $(P_i)_{i \in I}$ and the profile of adjusted priority rankings $(\pi_b^+)_{b \in B}$. A cadet pays the base cost for her branch assignment if either she has not declared willingness to pay the increased cost for her assigned branch or the increased cost capacity for the branch is already filled with cadets who have lower baseline priorities. With the exception of those who remain unmatched, all other cadets pay the increased cost for their branch assignments.

4.2 Shortcomings of the USMA-2020 Mechanism

Example 2 in Section 5.2 shows that the USMA-2020 mechanism fails both BRADSO-IC and elimination of strategic BRADSO, and Example 3 in Section 5.2 shows that it can admit detectable priority reversals even under its Bayesian Nash equilibrium outcomes. Before formally presenting these examples in the next section, we first describe how these failures already surfaced at the USMA in Fall 2019, paving the way for our collaboration with the Army.

Before a formal analysis of the USMA-2020 mechanism was carried out by our team, USMA leadership already recognized the possibility of detectable priority reversals under the USMA-2020 mechanism due to either failure of BRADSO-IC or presence of strategic BRADSO. For example, in a typical year, the number of cadets willing to BRADSO for traditionally oversubscribed branches like Military Intelligence greatly exceeded 25 percent of the branch's allocations. Therefore, by volunteering for BRADSO for an oversubscribed branch, some cadets could receive a priority upgrade even though they may not be charged for it, making detectable priority reversals a theoretical possibility. Moreover, unlike the detectable priority reversals under the USMA-2006 mechanism, some of these detectable priority reversals can affect cadet branch assignments, thereby making manual ex-post adjustments infeasible.

Failures of BRADSO-IC, elimination of strategic BRADSO, or presence of detectable priority reversals, especially when not manually corrected ex-post, could erode cadets' trust in the Army's branching process. Consider, for example, a comment from a cadet survey administered to the

high priority tier, but results from the simulation indicated the branch was very likely to extend contracts to medium priority cadets by the Engineer branch. As a result, cadets who volunteered to BRADSO for Engineer who were also placed in the high priority tier by the branch, faced a high probability of being charged BRADSOs under the USMA-2020 mechanism even though it was unlikely these cadets needed to BRADSO to branch Engineer.

Several open-ended survey comments from USMA cadets in the Class of 2020 mirrored USMA leadership's concern that continued use of the USMA-2020 mechanism would erode trust in the branching process. We present three additional comments articulating concerns related to the lack of BRADSO-IC, the presence of strategic BRADSO, and the dif culty of navigating a system with both shortcomings:

1) "Volunteering for BRADSO should only move you ahead of others if you are actually charged for BRADSO. By doing this, each branch will receive the most qualified people. Otherwise people who are lower in class rank will receive a branch over people that have a higher class rank which does not benefit the branch. Although those who BRADSO may be willing to serve longer, if they aren't charged then they can still leave after their 5 year commitment so it makes more sense to take the cadets with a higher OML."

2) "I think it is still a little hard to comprehend how the branching process works. For example, I do not know if I put a BRADSO for my preferred branch that happens to be very competitive, am I at a significantly lower chance of getting my second preferred if it happens to be something like engineers? Do I have to BRADSO now if I want engineers??? Am I screwing myself over by going for this competitive branch now that every one is going to try to beat the system????"

3) "Releasing the simulation just created chaos and panicked cadets into adding a BRADSO who otherwise wouldn't have."

4.3 USMA-2006 and USMA-2020 Mechanism in the Field

In this section, we use administrative data on cadet rankings, branch priorities, and capacities to investigate the performance of the USMA-2006 and USMA-2020 mechanisms. The data cover the West Point Classes of 2014 through 2021. Table 1 lists the capacity for each branch, the number of cadets who list the branch as their top choice, and the number of cadets who expressed a willingness to BRADSO for each branch for the Classes of 2020 and 2021. For the Class of 2020, 1,089 cadets participated in the branching process for 17 different branches. For the Class of 2021, 994 cadets participated in the branching process for 18 different branches.[20]

Figure 1 tabulates the incidence of BRADSO-IC failures, strategic BRADSO, and detectable priority reversals among USMA cadets across the USMA-2006 and USMA-2020 mechanism. For the USMA-2006 mechanism, we report the average across the Class of 2014 through Class of 2019.

[20] We successfully replicated the branch assignment for 99.2% of cadets in the Classes of 2014 through 2021. See Appendix B for details on our replication rates for each class.

Nearly four times as many cadets are part of BRADSO-ICs from the Class of 2020 (where the USMA-2020 mechanism was used) than earlier Classes from 2014 to 2019 (where USMA-2006 mechanism was used). Figure 1 shows about 22 cadets were part of BRADSO-IC failures under the USMA-2006 mechanism, while 85 cadets were part of BRADSO-IC failures under the USMA-2020 mechanism. Parallel to the incidences on BRADSO-IC failures, Figure 1 shows that nearly four times as many cadets are part of detectable priority reversals under the USMA-2020 mechanism than under the USMA-2006 mechanism (75 versus 20). It is not possible to have a strategic BRADSOs under the USMA-2006 mechanism. Figure 1 shows that 18 cadets in the Class of 2020 were part of strategic BRADSOs under the USMA-2020 mechanism. Importantly, these instances are not possible to remedy ex-post since that would require a change in branch assignments (rather than merely foregoing a BRADSO charge).

5 Single Branch Analysis

As with the USMA-2006 mechanism, truthful revelation of branch preferences is not a dominant strategy under the USMA-2020 mechanism, thereby making its analysis challenging. Fortunately, focusing on a simpler version of the model with a single branch is suf cient to illustrate and analyze the main challenges of the USMA-2020 mechanism. Focusing on this simpler model also offers a clear path to overcome these shortcomings, a path which is extended in Section 6 to the model in its full generality with multiple branches.

When there is a single branch b 2 B, there are only two preferences for any cadet i 2 I. The base cost contract(i, b, t^0) is by assumption preferred by cadet i to both its increased cost version (i, b, t^+) and also to remaining unmatched. Therefore, the only variation in cadet i's preferences depends on whether the increased cost contact(i, b, t^+) is preferred to remaining unmatched. For any cadet i 2 I, jQj = 2 When there is a single branch b 2 B, since

- indicating willingness to serve the increased cost t^+ under a quasi-direct mechanism can be naturally mapped to the preference relation where the increased cost contact (i, b, t^+) is acceptable, whereas

- not doing so can be naturally mapped to the preference relation where the increased cost contact (i, b, t^+) is unacceptable,

any quasi-direct mechanism can be interpreted as a direct mechanism. Therefore, unlike the general version of the model, the axioms of BRADSO-IC and elimination of strategic BRADSO are well-de ned for direct mechanisms when there is a single branch, and moreover they are both implied by strategy-proofness. [21]

[21] BRADSO-IC and elimination of strategic BRADSO together are equivalent to strategy-proofness when there is a single branch. Strategy-proofness of a single branch, c435 op5Tnon-m17 i8(eover)5siG -27aesinglact

5.1 Single-Branch Mechanism f^{BR} and Its Characterization

We next introduce a single-branch direct mechanism that is key for our analysis of the USMA-2020 mechanism. The main feature of this mechanism is its iterative subroutine (in Step 2), which then finalize Step 2 and proceed to Step 3.[22] In this case ℓ positions will be assigned at the increased cost t^+.

Otherwise, if

$$\left|\{ j \in J^{\ell} : (j, t^+) \; w_b^+ \; (i^{\ell+1}, t^0) \}\right| \geq \ell + 1,$$

then proceed to Step 2.$(\ell + 1)$, unless $\ell = q_b^+$, in which case finalize Step 2 and proceed to Step 3.

Step 3. Let Step 2.n be the final sub-step of Step 2 leading to Step 3. $\{i^1, \ldots, i^n\} \subseteq I^1$ is the set of cadets in I^1 who each lose their tentative assignment (b, t^0). For each cadet $i \in I^1 \setminus \{i^1, \ldots, i^n\}$, finalize the assignment of cadet i as $f_i^{BR}(\;) = (b, t^0)$.

For each cadet $i \in J^n$ with one of the n highest p_b-priorities in J^n, finalize the assignment of cadet i as $f_i^{BR}(\;) = (b, t^+)$

Example 1. (Mechanics of Mechanism φ^{BR}) There is a single branch b with $q_b^0 = 3$ and $q_b^+ = 3$. There are eight cadets, with their set given as $I = \{i^1, i^2, i^3, i^4, i^5, i^6, j^1, j^2\}$. The baseline priority order π_b is given as

$$i^6 \; \pi_b \; i^5 \; \pi_b \; i^4 \; \pi_b \; i^3 \; \pi_b \; i^2 \; \pi_b \; i^1 \; \pi_b \; j^1 \; \pi_b \; j^2,$$

and the BRADSO policy is the ultimate BRADSO policy $\overline{\omega}_b^+$. Cadet preferences are given as

$$(b, t^0) \succ_i (b, t^+) \succ_i \emptyset \qquad \text{for any } i \in \{i^1, i^3, i^5, j^1\}, \text{ and}$$
$$(b, t^0) \succ_i \emptyset \succ_i (b, t^+) \qquad \text{for any } i \in \{i^2, i^4, i^6, j^2\}.$$

We next run the procedure for the mechanism φ^{BR}.

Step 0: There are three regular positions. The three highest π_b-priority cadets in the set I are i^6, i^5, and i^4. Let $I^0 = \{i^4, i^5, i^6\}$, and finalize the assignments of cadets in I^0 as $\varphi^{BR}_{i^6}(\succ) = \varphi^{BR}_{i^5}(\succ) = \varphi^{BR}_{i^4}(\succ) = (b, t^0)$.

Step 1: There are three BRADSO-eligible positions. Three highest π_b-priority cadets in the set $I \setminus I^0$ are i^3, i^2, and i^1. Let $I^1 = \{i^1, i^2, i^3\}$, and the tentative assignment of each cadet in I^1 is (b, t^0). There is no need to relabel the cadets since cadet i^1 is already the lowest π_b-priority cadet in I^1, cadet i^2 is the second lowest π_b-priority cadet in I^1, and cadet i^3 is the highest π_b-priority cadet in I^1.

Step 2.0: The set of cadets in $I \setminus (I^0 \cup I^1) = \{j^1, j^2\}$ for whom the assignment (b, t^+) is acceptable is $\mathcal{J}^0 = \{j^1\}$. Since

$$\underbrace{\left|\left\{j \in \mathcal{J}^0 : (j, t^+) \; \overline{\omega}_b^+ \; (i^1, t^0)\right\}\right|}_{= |\mathcal{J}^0| = |\{j^1\}| = 1} \geq 1,$$

we proceed to Step 2.1.

Step 2.1: Since $(b, t^+) \succ_{i^1} \emptyset$, we have $\mathcal{J}^1 = \mathcal{J}^0 \cup \{i^1\} = \{i^1, j^1\}$. Since

$$\underbrace{\left|\left\{j \in \mathcal{J}^1 : (j, t^+) \; \overline{\omega}_b^+ \; (i^2, t^0)\right\}\right|}_{= |\mathcal{J}^1| = |\{i^1, j^1\}| = 2} \geq 2,$$

we proceed to Step 2.2.

Step 2.2: Since $\emptyset \succ_{i^2} (b, t^+)$, we have $\mathcal{J}^2 = \mathcal{J}^1 = \{i^1, j^1\}$. Since

$$\underbrace{\left|\left\{j \in \mathcal{J}^2 : (j, t^+) \; \overline{\omega}_b^+ \; (i^3, t^0)\right\}\right|}_{= |\mathcal{J}^2| = |\{i^1, j^1\}| = 2} = 2,$$

we finalize Step 2 and proceed to Step 2.3.

Step 3: Step 2.2 is the last sub-step of Step 2. Therefore two lowest π_b-priority cadets in I^1, i.e cadets i^1 and i^2, lose their tentative assignments of (b, t^0). In contrast, the only remaining cadet in the set $I^1 \setminus \{i^1, i^2\}$, i.e cadet i^3 maintains her tentative assignment, which is finalized as $\varphi^{BR}_{i^3}(\succ) = (b, t^0)$.

The two highest priority cadets in ϑ^2 are i^1 and j^1. Their assignments are finalized as $\phi^{BR}_{i^1}(\ ) = \phi^{BR}_{j^1}(\ ) = (b, t^+)$. Assignments of the remaining cadets i^2 and j^2 are finalized as $\emptyset$. The final allocation is:

$$\phi^{BR}(\ ) = \begin{pmatrix} i^1 & i^2 & i^3 & i^4 & i^5 & i^6 & j^1 & j^2 \\ (b,t^+) & \emptyset & (b,t^0) & (b,t^0) & (b,t^0) & (b,t^0) & (b,t^+) & \emptyset \end{pmatrix}.$$

Our first result shows that when there is a single branch the direct mechanism ϕ^{BR} is the only mechanism that satisfies our main desiderata.

Theorem 1. *Suppose there is a single branch b. Fix a baseline priority order $\pi_b \in \Pi$ and a BRADSO policy $\omega_b^+ \in \Omega_b^+$. A direct mechanism ψ satisfies*

1. *individual rationality,*
2. *non-wastefulness,*
3. *enforcement of the BRADSO policy,*
4. *BRADSO-IC, and*
5. *has no priority reversals,*

if and only if $\psi = \phi^{BR}$.

5.2 Equilibrium Outcomes under the USMA-2020 Mechanism

While the USMA-2020 mechanism is not a direct mechanism in general, when there is a single branch it can be interpreted a direct mechanism. In this case, for any cadet $i \in I$ the first part of the strategy space $S_i = \mathcal{P} \quad 2^B$ becomes redundant, and the second part simply solicits whether branch b is acceptable by cadet i or not (analogous to a direct mechanism).

Our next result shows that when there is a single branch the truthful outcome of the direct mechanism ϕ^{BR} is the same as the unique Nash equilibrium outcome of the mechanism ψ^{2020}.

Proposition 1. *Suppose there is a single branch b. Fix a baseline priority order $\pi_b \in \Pi$, a BRADSO policy $\omega_b^+ \in \Omega_b^+$, and a preference profile $\ \in \Theta^{|I|}$. Then the strategic-form game induced by the mechanism (S^{2020}, ψ^{2020}) has a unique Nash equilibrium outcome that is equal to the allocation $\phi^{BR}(\ )$.*[23]

Caution is needed when interpreting Proposition 1; if interpreted literally, this result can be misleading. What is more consequential for Proposition 1 is not the result itself, but rather its proof which constructs the equilibrium strategies of cadets. The proof provides insight into why

[23] Using the terminology of the implementation theory, this result can be alternatively stated as follows: When there is a single branch, the mechanism (S^{2020}, ψ^{2020}) implements the allocation rule ϕ^{BR} in Nash equilibrium. See Maskin and Sjöström (2002) and Jackson (2001) for surveys of implementation theory.

the failure of BRADSO-IC, the presence of strategic BRADSO, and the presence of detectable priority reversals are all common phenomena under the real-life implementation of the USMA-2020 mechanism (despite the outcome equivalence suggested by Proposition 1).

Given the byzantine structure of the Nash equilibrium strategies even with a single branch, it is perhaps not surprising that reaching such a well-behaved Nash equilibrium is highly unlikely to be observed under the USMA-2020 mechanism. The following example illustrates the knife-edge structure of the Nash equilibrium strategies under the USMA-2020 mechanism.

Example 2. (Knife-Edge Nash Equilibrium Strategies)

To illustrate how challenging it is for the cadets to figure out their best responses under the USMA-2020 mechanism, we present two scenarios. The scenarios differ from each other minimally, but cadet best responses differ dramatically. Our first scenario is same as the one we presented in Example 1.

Scenario 1: There is a single branch b with $q_b^0 = 3$ and $q_b^+ = 3$. There are eight cadets, $I = \{i^1, i^2, i^3, i^4, i^5, i^6, j^1, j^2\}$. The baseline priority order π_b is given as

$$i^6 \; \pi_b \; i^5 \; \pi_b \; i^4 \; \pi_b \; i^3 \; \pi_b \; i^2 \; \pi_b \; i^1 \; \pi_b \; j^1 \; \pi_b \; j^2 \quad \text{and}$$

and the BRADSO policy is the ultimate BRADSO policy $\overline{\omega}_b^+$. Cadet preferences are

$$(b,t^0) \succ_i (b,t^+) \succ_i \emptyset \qquad \text{for any } i \in \{i^1, i^3, i^5, j^1\}, \text{ and}$$
$$(b,t^0) \succ_i \emptyset \succ_i (b,t^+) \qquad \text{for any } i \in \{i^2, i^4, i^6, j^2\}.$$

Let s^* be a Nash equilibrium strategy for Scenario 1 under the USMA-2020 mechanism. Recall that when there is a single branch b, the strategy space for each cadet $i \in I$ is simply $S_i = \{b, \emptyset\}$. We construct the Nash equilibrium strategies in several phases.

Phase 1: Consider cadets i^1 and j^1, each of whom prefers the increased-cost assignment (b,t^+) to remaining unmatched. Since there are six positions altogether and there are five higher π_b-priority cadets than either of these two cadets, at most one of them can receive a position (at any cost) unless each of them submit a strategy of b. And if one of them submit a strategy of $\emptyset$, the other one has a best response strategy of b assuring a position at the increased cost rather than remaining unmatched. Hence, $s^*_{i^1} = s^*_{j^1} = b$ at any Nash equilibrium.

Phase 2: Consider cadet j^2 who prefers remaining unmatched to the increased-cost assignment (b,t^+). Since she is the lowest π_b-priority cadet, she cannot receive an assignment of (b,t^0) regardless of her strategy. In contrast, she can guarantee remaining unmatched with a strategy of $s_{j^2} = \emptyset$. While this does not at this point rule out a strategy of $s_{j^2} = \emptyset$ at Nash equilibrium (just yet), it means $\varphi^{2020}_{j^2}(s^*) = \emptyset$.

Phase 3: Consider cadet i^2 who prefers remaining unmatched to the increased-cost assignment (b,t^+). She is the fifth highest π_b-priority cadet, so she secures a cadet j^2 is remaining unmatched from Phase 2, and therefore there cannot be three cadets with lower p_b-priority who receive an assignment of (b, t^+). But since cadet j^2 prefers remaining unmatched to the increased-cost assignment (b, t^+), she cannot receive an assignment of (b, t^+) at Nash equilibria. Hence, her Nash equilibrium strategy is $s_{j^2} = Æ$, and her Nash equilibrium assignment is $j^{2020}_{j^2}(s) = Æ$

Phase 4: Consider the remaining cadets i^3, i^4, i^5 and i^6. Since cadets i^2 and j^2 have to remain unmatched (from Phases 2 and 3) at Nash equilibria, they each receive a position at Nash equilibrium. Since only the two cadets i^{050}

a strategy of b, this assures that exactly three positions will be assigned at the increased cost t^+. Therefore a strategy of f $s_{i^2} = b$ assures assures cadet^2 an assignment of (b, t^+), which cannot happen at Nash equilibrium. Therefore, $s^0_{i^2} =$ Æ and $j^{2020}_{i^2}(s^0) =$ Æ. This not only assures that $j^{2020}_{i^3}(s^0) = j^{2020}_{i^1}(s^0) = j^{2020}_{j^1}(s^0) = (b, t^+)$, but it also means that $s^0_{j^2} = b$ at Nash equilibrium, for otherwise with two lower p_b-priority cadets with strategies of Æ, cadet i^3 would have an incentive

Suppose there is a single branch b with $q_b^0 = q_b^+ = 1$ and three cadets i_1, i_2, and i_3. The baseline priority order p_b is such that

$$i_1 \; p_b \; i_2 \; p_b \; i_3,$$

and the BRADSO policy w_b^+ is the ultimate BRADSO policy $\overline{w}_b^+$.

Each cadet has a utility function that is drawn from a distribution with the following two elements, u and v, where:

$$u(b,t^0) = 10,\ u(\emptyset) = 8,\ u(b,t^+) = 0, \quad \text{and} \quad v(b,t^0) = 10,\ v(b,t^+) = 8,\ v(\emptyset) = 0.$$

Let us refer to cadets with a utility function $u(.)$ as type 1 and cadets with a utility function $v(.)$ as type 2. All cadets have a utility of 10 for their first choice assignment of (b,t^0), a utility of 8 for their second choice assignment, and a utility of 0 for their last choice assignment. For type 1 cadets, the second choice is remaining unmatched whereas for type 2 cadets the second choice is receiving a position at the increased cost t^+. Suppose each cadet can be of the either type with a probability of 50 percent, and they are all expected utility maximizers.

The unique Bayesian Nash equilibrium s under the incomplete information game induced by the USMA-2020 mechanism is, for any cadet $i \in \{i_1, i_2, i_3\}$,

$$s_i = \begin{cases} \emptyset & \text{if cadet } i \text{ is of type 1, and} \\ b & \text{if cadet } i \text{ is of type 2.} \end{cases}$$

That is, truth-telling is the unique Bayesian Nash equilibrium strategy for each cadet. However, this unique Bayesian Nash equilibrium strategy results in detectable priority reversals whenever either

1. cadet i_1 is of type 1 and cadets i_2, i_3 32

unnecessary. Indeed, some of the cadets indicated the need for a system that would allow them to rank order branch-cost pairs. One cadet wrote:

> “ [. . .] I believe that DMI (Department of Military Instruction) could elicit a new type of ranking list. Within my proposed system, people could add to the list of 17 branches BRADSO slots and rank them within that list. For example: AV (Aviation) > IN (Infantry)

The native linear order ω_b^0 simply mirrors the baseline priority order π_b, and prioritizes cadet-cost pairs in $I \times T$ as the cadet of the pair is prioritized under the baseline priority order π_b, while giving higher priority to the base cost t^0 over the increased cost t^+ for any given cadet.

Under the COM-BRADSO mechanism, each branch $b \in B$ relies on the following choice rule to select a set of contracts from any set of contracts viable for branch b.

> **Choice Rule C_b^{BR}**
>
> For any set of contracts $X \subseteq \mathcal{X}_b$ that is viable for branch b,
>
> Step 1. If there are less than q_b^0 contracts in X with distinct cadets, then choose all contracts in X with the base cost t_0 and terminate the procedure. In this case $C_b^{BR}(X) = \{x \in X : t(x) = t^0\}$.
>
> Otherwise, let X_1 be the set of q_b^0 highest ω_b^0-priority contracts in X with distinct cadets.[25] Pick contracts in X_1 and proceed to Step 2.
>
> Step 2. The set of contracts under consideration for this step is
>
> $$Y = \left\{ x \in X \setminus X_1 : \left(i(x), b, t^0\right) \notin X_1 \right\}.$$
>
> If there are less than q_b^+ contracts in Y with distinct cadets, then pick all contracts in Y with the base cost t^0 and terminate the procedure. In this case $C_b^{BR}(X) = X_1 \cup \{x \in Y : t(x) = t^0\}$.
>
> Otherwise, let X_2 be the set of q_b^+ highest ω_b^+-priority contracts in Y with distinct cadets. Pick contracts in X_2 and terminate the procedure. In this case $C_b^{BR}(X) = X_1 \cup X_2$.

Intuitively, the choice rule C_b^{BR} relies on the native priority order ω_b^0 for the first q_b^0 positions, and on the BRADSO policy ω_b^+ for the last q_b^+ positions.

Observe that all increased cost contracts are selected in Step 2 of the choice rule C_b^{BR}. Therefore, an increase in the BRADSO cap means using the native priority order ω_b^0 for fewer positions and the BRADSO policy ω_b^+ for more positions, thereby weakly increasing the number of increased-cost contracts selected by the choice rule C_b^{BR}. Moreover, since the increased-cost contracts receive weakly higher priorities when the BRADSO policy becomes more effective at branch b, such a change in the BRADSO policy also weakly increases the number of increased-cost contracts selected by the choice rule C_b^{BR}. We state these two observations in the following result.

Proposition 2. *For any branch $b \in B$ and set of contracts $X \subseteq \mathcal{X}_b$ viable for branch b,*

1. *the higher the BRADSO cap q_b^+ is the weakly higher is the number of increased cost contracts accepted under $C_b^{BR}(X)$, and*

[25] Since X is viable and ω_b^0 is the native priority order, all contracts in X_1 has the base cost t^0.

2. the more effective the BRADSO policy w_b^+ is the weakly higher is the number of increased cost contracts accepted under $C_b^{BR}(X)$.

We are ready to introduce the mechanism central to the Army's 2021 Branching reform. For a given list of BRADSO policies $(w_b^+)_{b \in B}$, let $C^{BR} = (C_b^{BR})_{b \in B}$ denote the list of branch-specific choice rules defined above. COM-BRADSO mechanism is a direct mechanism where each cadet reports her preferences over $B \times T \cup \{\emptyset\}$. Therefore, the strategy space for each cadet $i \in I$ is

$$S_i^{COM-BR} = Q.$$

The outcome function f^{COM-BR} for the COM-BRADSO mechanism is given through the following procedure.

Cumulative Offer Mechanism under C^{BR}

Fix a linear order of cadets $\pi \in \Pi$.[26] For a given profile of cadet preferences $\succ = (\succ_i)_{i \in I} \in Q^{|I|}$, cadets propose their acceptable contracts to branches in a sequence of steps $\ell = 1, 2, \ldots$:

Step 1. Let $i_1 \in I$ be the highest π-ranked cadet who has an acceptable contract. Cadet $i_1 \in I$ proposes her most preferred contract $x_1 \in X_{i_1}$ to branch $b(x_1)$. Branch $b(x_1)$ holds x_1 if $x_1 \in C^{BR}_{b(x_1)}(\{x_1\})$ and rejects x_1 otherwise. Set $A^2_{b(x_1)} = \{x_1\}$ and set $A^2_{b'} = \emptyset$ for each $b' \in B \setminus \{b(x_1)\}$; these are the sets of contracts available to branches at the beginning of step 2.

Step ℓ. Let $i_\ell \in I$ be the highest π-ranked cadet for whom no contract is currently held by any branch, and let $x_\ell \in X_{i_\ell}$ be her most preferred acceptable contract that has not yet been rejected. Cadet i_ℓ proposes contract x_ℓ to branch $b(x_\ell)$. Branch $b(x_\ell)$ holds the contracts in $C^{BR}_{b(x_\ell)}(A^\ell_{b(x_\ell)} \cup \{x_\ell\})$ and rejects all other contracts in $A^\ell_{b(x_\ell)} \cup \{x_\ell$ $_{b(x_\ell}$

Our final and main theoretical result shows COM-BRADSO is the only mechanism that satisfies all our desiderata.

Theorem 2. Fix a profile of baseline priority orders $(\succ_b)_{b \in B} \in \mathcal{P}$ and a profile of BRADSO policies $(\omega_b^+)_{b \in B} \in \times_{b \in B} \Omega_b^+$. A direct mechanism φ satisfies

1. individual rationality,
2. non-wastefulness,
3. enforcement of the BRADSO policy,
4. strategy-proofness, and
5. has no priority reversals,

if and only if φ is the COM-BRADSO mechanism φ^{COM-BR}.

Apart from singling out the COM-BRADSO mechanism as the unique mechanism that satisfies our desiderata, to the best of our knowledge Theorem 2 is the first joint characterization of an allocation mechanism (i.e. the cumulative offer process) together with a specific choice rule C_b^{BR} for each branch $b \in B$.[27] In our application, in addition to the standard axioms of individual rationality, non-wastefulness, lack of priority reversals, and strategy-proofness, the axiom of enforcement of the BRADSO policy (wh [(w)]TJ/F143 7.970 Tf 8.571 4.438 Td [(+)]TJ/F131 7.970

Class of 2021 confirms that this flexibility was used by cadets. Figure 2 provides details on the extent to which cadets did not rank a branch with increased cost immediately after the branch at base cost. For each of 994 cadet first branch choices, 272 cadets rank that branch with BRADSO as their second choice and 36 cadets rank that branch with BRADSO as their third choice or lower. These 36 cadets would not have been able to express this preference under the message space of a quasi-direct mechanism like the USMA-2006 mechanism or the USMA-2020 mechanism. When we consider the next branch on a cadet's rank order list, cadets also value the flexibility of the new mechanism. For the branch that appears next on the rank order list, 78 cadets rank that branch with BRADSO as their immediate next highest choice and 24 cadets rank that branch with BRADSO two or more places below on their rank order list. These 24 cadets also would not have been able to express this preference under a quasi-direct mechanism.

The fact that COM-BRADSO is a strategy-proof mechanism which elicits rankings over branch-price pairs allows us to compare outcomes under the USMA-2006 and USMA-2020 mechanisms with knowledge of the underlying branch-price preference relationship. In Figure 1, we could rankings of branches and learned about their assignment took place. After observing their dry-run assignment, cadets were allowed to submit a final set of rankings under USMA-2020, and therefore had the opportunity to revise their strategies in response to this feedback. Figure 4 tabulates strategic BRADSOs, BRADSO-IC failures, and detectable priority reversals under indicative and final preferences. Final preferences result in fewer strategic BRADSOs, BRADSO-IC failures, and detectable priority reversals. This pattern is consistent with some cadets responding to the dry-run by ranking branch choices in response to these issues.

In general, cadets form their preferences over branches over time as they acquire more information about branches and their own tastes. Therefore, the change documented in Figure 4 may simply reflect general preference formation from acquiring information about branches, and not revisions to preferences in response to the specific mechanism. We briefly investigate this possibility by looking at the presence of strategic BRADSOs, BRADSO-IC failures, and priority reversals using data on the indicative and final preferences from the Class of 2021. This class participated in the strategy-proof COM-BRADSO mechanism. We take indicative and final cadet preferences under COM-BRADSO and construct truthful strategies, following the approach described above, for the USMA-2020 mechanism. Figure 5 shows that with preferences constructed from a strategy-proof mechanism, there are only modest differences in strategic BRADSOs, BRADSO-IC failures,

> A key question the Army considered when designing this year's mechanism was how much in uence to give cadets who are willing to BRADSO. If every cadet who volunteers to BRADSO can gain priority, or "jump" above, every cadet who did not volunteer to BRADSO, then that could improve Army retention through more cadets serving an additional three years, but it could also result in more cadets being assigned to branches that do not prefer them.

The comparative static results in Proposition 2 in Section 6.1 motivate our empirical analysis of different BRADSO policies. While the results on the BRADSO collected given in Proposition 2 hold for a given branch, in theory they may not hold in aggregate across all branches under COM-BRADSO.[28] However, as we show next, the comparative static properties do hold in our simulations with the Class of 2021 data for several BRADSO policies.

The Army considered three BRADSO policies: the ultimate BRADSO policy and two tiered BRADSO policies. Under BRADSO-2020, a cadet who expressed a willingness to sign a BRADSO contract only obtained priority over other cadets who had the same categorical branch rating. Under BRADSO-2021, a cadet who expressed a willingness to sign a BRADSO contract obtained higher priority over all other cadets if she was in the medium or high category. To illustrate the trade-off between talent alignment and retention, Figure 6 uses preferences from the Class of 2021 and re-runs the COM-BRADSO mechanism under these three BRADSO policies for different levels of BRADSO cap q_b^+, where q_b^+ is expressed as a percentage of q_b, the total number of positions for branch b.

To measure the effects of BRADSO policies on BRADSOs collected, Figure 6 shows how the number of BRADSOs charged increases with q_b^+ and with the closeness of the BRADSO policy to the ultimate BRADSO policy. That is, for a given q_b^+ the BRADSO-2021 policy results in more BRADSOs charged than BRADSO-2020 policy, but fewer BRADSOs charged than the ultimate BRADSO policy. When the BRADSO cap is small, there is relatively little difference between BRADSO policies. For example, when the BRADSO cap is 15% of slots, 55 BRADSOs are charged under the ultimate BRADSO, 47 BRADSOs are charged under BRADSO-2021, and 38 BRADSOs policy and increase the BRADSO cap, q_b^+, from 25 to 35 percent. These are both policies that increase the power of BRADSO. However, USMA decided against adopting the ultimate BRADSO policy because branches remained opposed to giving more BRADSO power to low tier cadets.

7 Conclusion

In July 2019, the US Army implemented sweeping changes to the Army's Talent-Based Branching Program by adopting the USMA-2020 mechanism for the West Point, or USMA, Class of 2020. The impetus for this change was to give Army branches greater influence and to ultimately assign cadets to better fitting branches. However, the USMA-2020 mechanism retained the same restricted strategy space as the previous USMA-2006 mechanism. The performance of the USMA-2020 mechanism made several underlying issues more apparent.

Our paper describes these reforms and shows how they facilitated the adoption of a cumulative offer mechanism for the Class of 2021. Our main result is that the cumulative offer mechanism with a particular choice function is the only mechanism that satisfies intuitive criteria, all formulating the Army's objectives. We also formally and empirically study the USMA-2020 mechanism. That investigation provides insights into the perverse incentives in this mechanism and why these challenges became difficult to ignore for the Class of 2020.

When it was first formulated in Sönmez and Switzer (2013), cadet-branch matching became the first real-life application of the matching with contracts framework with a non-trivial role for the contractual terms. Our work builds on foundational theory by Kelso and Crawford (1982), Hatfield and Milgrom (2005), and Hatfield and Kojima (2010) and applied theory papers by Sönmez and Switzer (2013) and Sönmez (2013). This sequence of papers opened the door to influence mechanisms deployed in the field, and eventually led to the redesign of USMA's mechanism. In this respect, we contribute to a market design literature where abstract theoretical models, which are often not contemplated in terms of particular applications, go on to have practical applications and ultimately influence real-world mechanisms. We hope the chronology of the military's reform which links theory to practice follows the model of other market design applications, such as for the medical match, spectrum auctions, school assignment, kidney exchange, internet advertising, and course assignment.[30] Moreover, after the adoption of the cumulative offer mechanism at the Israeli Psychology Master's Match (Hassidim, Romm, and Shorrer, 2017), the Army's use of the COM-BRADSO mechanism is, as far as we know, the second field application of matching with

SOs are consecutive, and also considered different assumptions on the prevalence of non-consecutive BRADSOs. These assumptions are not needed when cadets can rank branch-price pairs in a strategy-proof mechanism.

[30] For the medical match, see Gale and Shapley (1962), Roth (1982), and Roth and Peranson (1999). For package auctions, see Kelso and Crawford (1982), Demange, Gale, and Sotomayor (1986), Milgrom (2000), Ausubel and Milgrom (2003), Milgrom and Segal (2017), and Milgrom and Segal (2020). For school assignment, see Gale and Shapley (1962), Balinski and Sönmez (1999), Abdulkadiroğlu and Sönmez (2003), Pathak and Sönmez (2008), and Abdulkadiroğlu, Pathak, and Roth (2009). For kidney exchange, see Shapley and Scarf (1974), Abdulkadiroğlu and Sönmez (1999), Roth, Sönmez, and Ünver (2004) and Roth, Sönmez, and Ünver (2005). For internet advertising, see Shapley and Shubik (1971), Edelman, Ostrovsky, and Schwarz (2007), and Varian (2006). For course allocation, see Varian (1974), Sönmez and Ünver (2010), Budish (2011), Budish and Cantillon (2012), and Budish, Cachon, Kessler, and Othman (2017).

contracts.

While the Army initially resisted reforms to the USMA branching process, the challenges due to failures of certain principles formalized by our axioms led the Army to partner with us to x these challenges. The Army sought a mechanism that not only promoted retention and talent alignment as USMA-2020 did, but that was also incentive compatible. The desire for incentive compatibility was partly to build cadets' trust in Army labor markets (Garcia, 2020), and partly to obtain truthful information on cadet preferences. The latter objective is particularly important for Army efforts to understand and address the lack of minority representation in branches like Infantry and Armor, branches that produce a disproportionate share of Army generals (Briscoe, 2013; Kofoed and mcGovney, 2019). In that sense, reform shows the practical relevance and power of the matching with contracts framework, as well as the importance of building mechanisms with

References

Abdulkadiroglu, A., P. A. Pathak, and A. E. Roth

COLARRUSO, M., D. S. LYLE, AND C. WARDYNSKI (2010): “Towards a U.S. Army Of cer Corps Strategy for Success: Retaining Talent,” Unpublished manuscript, Strategic Studies Institute.

COLARUSSO, M. J., K. G. HECKEL, D. S. LYLE, AND W. L. SKIMMYHORN (2016): “Starting Strong: Talent-Based Branching of Newly Commissioned US Army Of cers (Of cer Corps Strategy, Volume 9),” Discussion paper, Army War College-Strategic Studies Institute Carlisle United States.

COWGILL, B., J. DAVIS, B. P. MONTAGNES, AND P. PERKOWSKI (2021): “Matching for Strategic Organizations: Theory and Empirics from Internal Labor Markets,” *Available at SSRN*.

CRAWFORD, V. P. (2008): “The Flexible-Salary Match: A Proposal to Increase the Salary Flexibility of the National Resident Matching Program,” *Journal of Economic Behavior and Organization*, 66, 149–160.

DEMANGE, G., D. GALE, AND M. A. O. SOTOMAYOR (1986): “Multi-Item Auctions,” *Journal of Political Economy*, 94(4), 863–872.

DOD (2020): “Population Representation in the Military Services: Fiscal Year 2018,” Of ce of the Under Secretary of Defense (Personnel and Readiness).

DUR, U., S. D. KOMINERS, P. A. PATHAK, AND T. SÖNMEZ (2018): “Reserve Design: Unintended Consequences and the Demise of Boston?s Walk Zones,” *Journal of Political Economy*, 126(6), 2457–2479.

DUR, U., P. A. PATHAK, AND T. SÖNMEZ (2020): “Explicit vs. Statistical Preferential Treatment in Af rmative Action: Theory and Evidence from Chicago's Exam Schools,” *Journal of Economic Theory*, 187, 104996.

ECHENIQUE, F. (2012): “Contracts vs. Salaries in Matching,” *American Economic Review*, 102(1), 594–601.

ECHENIQUE, F., AND B. YENMEZ (2015): “How to Control Controlled School Choice?,” *American Economic Review*, 105(8), 2679–2694.

EDELMAN, B., M. OSTROVSKY, AND M. SCHWARZ (2007): “Internet Advertising and the Generalized Second-Price Auction: Selling Billions of Dollars Worth of Keywords,” *American Economic Review*, 97(1), 242–259.

GALE, D., AND L. S. SHAPLEY (1962): “College Admissions and the Stability of Marriage,” *American Mathematical Monthly*, 69, 9–15.

GARCIA, J. (2020): “New innovations improve branching process for cadets, branches,” *Pointer-View*, September 17.

——— (2020): "Clock Auctions and Radio Spectrum Reallocation," *Journal of Political Economy*, 128(1).

MILGROM, P. R. (2000): "Putting Auction Theory to Work: The Simultaneous Ascending Auction," *Journal of Political Economy*, 108, 245–272.

MOUNT, K., AND S. REITER (1974): "The Information Size of Message Spaces," *Journal of Economic Theory*, 28, 1–28.

O'CONNOR, B. (2019): "Assigning branches to cadets takes on new system," https://www.army.mil/article/227124/assigning_branches_to_cadets_takes_on_new_system, Last Accessed: April 29, 2021.

PATHAK, P. A., AND T. SÖNMEZ (2008): "Leveling the Playing Field: Sincere and Sophisticated Players in the Boston Mechanism," *American Economic Review*, 98(4), 1636–1652.

——— (2013): "School Adm0eKl24

SHAPLEY, L., AND M. SHUBIK (1971): "The Assignment Game I: The core," *International Journal of Game Theory*, 1(1), 111–130.

SÖNMEZ, T. (2013): "Bidding for Army Career Specialties: Improving the ROTC Branching Mechanism," *Journal of Political Economy*, 121(1), 186–219.

SÖNMEZ, T., AND T. SWITZER (2013): "Matching with (Branch-of-Choice) Contracts at the United States Military Academy," *Econometrica*, 81(2).

SÖNMEZ, T., AND M. U. ÜNVER (2010): "Course Bidding at Business Schools," *International Eco-*

q_b^+-lowest π_b-priority cadets among these awardees are only tentative. Step 2 of the procedure for mechanism φ^{BR} ensures that, if any cadet $j \in I$ loses her tentative assignment (b,t^0) from Step 1, then any cadet $i \in I$ who receives an assignment of (b,t^+) is such that $(i,t^+)\ \omega_b^+\ (j,t^0)$. Therefore,

$$\left.\begin{array}{l}\varphi_i^{BR}(\succ) = (b,t^+), \text{ and}\\ (b,t^0) \succ_j \varphi_j^{BR}(\succ)\end{array}\right\} \implies (i,t^+)\ \omega_b^+\ (j,t^0). \tag{3}$$

Moreover, Step 2 of the same procedure also ensures that, for any $\ell \in \{1,\ldots,q_b^+\}$, the ℓ^{th} lowest π_b-priority cadet i_ℓ with a tentative assignment of (b,t^0) cannot maintain this tentative assignment, for as long as there are at least ℓ lower π_b-priority cadets who are both willing to pay the increased cost t^+ and also able to "jump ahead of" the cadet i_ℓ through the BRADSO policy. Therefore,

$$\left.\begin{array}{l}\varphi_j^{BR}(\succ) = (b,t^0),\\ (b,t^+) \succ_i \varphi_i^{BR}(\succ), \text{ and}\\ (i,t^+)\ \omega_b^+\ (j,t^0)\end{array}\right\} \implies \left|\{i' \in I : \varphi_{i'}^{BR}(\succ) = (b,t^+)\}\right| = q_b^+. \tag{4}$$

Relations (3) and (4) imply that mechanism φ^{BR} satisfies enforcement of the BRADSO policy.

Uniqueness: We next show that mechanism φ^{BR} is the only mechanism that satisfies all five axioms.

Let the direct mechanism ψ satisfy individual rationality, non-wastefulness, BRADSO-IC, enforcement of the BRADSO policy, and has no priority reversals. We want to show that $\psi(\succ) = \varphi^{BR}(\succ)$.

If there are less than or equal to q cadets for whom the assignment (b,t^0) is acceptable under the preference profile $\succ$, all such cadets must receive an assignment of (b,t^0) by individual rationality, non-wastefulness, and BRADSO-IC. Since this is also the case under the allocation $\varphi^{BR}(\succ)$, the result holds immediately for this case.

Therefore, w.l.o.g assume that there are strictly more than q cadets for whom the assignment (b,t^0) is acceptable under the preference profile $\succ$. Let I^0 be the set of q_b^0 highest π_b-priority cadets in I. By non-wastefulness, all positions are assigned under $\psi(\succ)$. Since at most q_b^+ positions can be awarded at the increased cost t^+, at least q_b^0 positions has to be allocated at the base cost t^0. Therefore,

$$\text{for any } i \in I^0, \quad \psi_i(\succ) = (b,t^0) = \varphi_i^{BR}(\succ) \tag{5}$$

by lack of priority reversals.

Let I^1 be the set of q_b^+ highest π_b-priority cadets in $I \setminus I^0$. Relabel the cadets in the set I^1 so that for any $\ell \in \{1,\ldots,q_b^+\}$, cadet i_ℓ is the ℓ^{th}-lowest π_b-priority cadet in I^1. Let

$$J^0 = \{j \in I \setminus (I^0 \cup I^1) : (b,t^+) \succ_j \emptyset\}.$$

By individual rationality and the lack of priority reversals,

$$\text{for any } i \in I \setminus (I^0 \cup I^1 \cup J^0), \qquad j_i(\ ) = \emptyset = f_i^{BR}(\ ). \tag{6}$$

By relations (5) and (6), the only set of cadets whose assignments are yet to be determined under $j(\ )$ are cadets in $I^1 \cup J^0$. Moreover, by the lack of priority reversals, cadets in J^0 can only receive a position at the increased cost t^+. That is,

$$\text{for any } j \in J^0, \qquad j_j(\ ) \neq (b, t^0). \tag{7}$$

For the next phase of our proof, we will rely on the sequence of individuals $i^1, \ldots, i^{q_b^+}$ and the sequence of sets $J^0, J^1, \ldots$ that are constructed for the Step 2 of the mechanism f^{BR}. Here individual i^1 is the q^{th} highest p_b-priority cadet in set I, cadet i^2 is the $(q\ \ 1)^{th}$ highest p_b-priority cadet in set I, and so on. The starting element of the second sequence is $J^0 = \{ j \in I \setminus ($ lack of priority reversals

Relations (5), and (11) imply $\varphi(\ ) = \varphi^{BR}(\ )$, completing the proof for Case 1.

Case 2. $n \in \{1, \ldots, q_b^+ \quad 1\}$

For this case, by the mechanics of the Step 2 of the mechanism φ^{BR}, we have

$$\text{for any } \ell \in \{1, \ldots, n\}, \qquad j \in J^{\ell \ 1} : (j, t^+) \ \omega_b^+ \ (i^{\ell}, t^0) \qquad \ell, \tag{12}$$

and

$$j \in J^n : (j, t^+) \ \omega_b^+ \ (i^{n+1}, t^0) \quad = n. \tag{13}$$

Since mechanism φ satisfies condition (2) of the axiom enforcement of the BRADSO policy, the lack of priority reversals and relation 12 imply

$$\text{for any } i \in \{i^1, \ldots, i^n\}, \qquad \varphi_i(\ ) \neq (b, t^0). \tag{14}$$

Therefore, by non-wastefulness and relations (5), (6), (7), and (14), at least n positions must be assigned at the increased cost t^+.

Moreover, since mechanism φ satisfies non-wastefulness, lack of priority reversals, and condition (1) of the axiom enforcement of the BRADSO policy, relation (13) implies

$$\text{for any } i \in \{i^{n+1}, \ldots, i^{q_b^+}\}, \qquad \varphi_i(\ ) \in \quad (b, t^0), (b, t^+) \quad . \tag{15}$$

But since φ satisfies individual rationality, relation (15) implies that $\varphi_i(\ ) = (b, t^0)$ for any $i \in \{i^{n+1}, \ldots, i^{q_b^+}\}$ with $\emptyset \quad_i (b, t^+)$. Furthermore for any $i \in \{i^{n+1}, \ldots, i^{q_b^+}\}$ with $(b, t^+) \quad_i \emptyset$, instead reporting the fake preference relation $\ _i^0 \in Q$ with $\emptyset \ _i^0 (b, t^+)$ would guarantee cadet i an assignment of $\varphi_i(\ _i, \ _i^0) = (b, t^0)$ due to the same arguments applied for the economy $(\ _i, \ _i^0)$, and therefore by BRADSO-IC these cadets too must receive an assignment of (b, t^0) each. Hence

$$\text{for any } i \in \{i^{n+1}, \ldots, i^{q_b^+}\}, \qquad \varphi_i(\ ) = (b, t^0) = \varphi_i^{BR}(\ ). \tag{16}$$

Since we have already shown that at least n positions must be assigned at an increased cost of t^+, relation (16) implies that exactly n positions must be assigned this cost, and therefore for any cadet $j \in J^n$ who is one of the n highest p_b-priority cadets in J^n,

$$\varphi_j(\ ) = (b, t \ \text{with}$$

Since mechanism φ satisfies condition (2) of the axiom enforcement of the BRADSO policy, relation 18 implies

$$\text{for any } i \in \underbrace{\{i^1, \ldots, i^{q_b^+}\}}_{= I^1}, \qquad \varphi_i(\ ) \neq (b, t^0). \tag{19}$$

Therefore, by non-wastefulness and the lack of priority reversals, exactly q_b^+ positions must be assigned at the increased cost t^+. Hence for any cadet $j \in J^{q_b^+}$ who is one of the q_b^+ highest p_b-priority cadets in $J^{q_b^+}$,

$$\varphi_j(\ ) = (b, t^+) = f_i^{BR}(\ ) \tag{20}$$

by elimination of priority reversals.

Relations (5) and (20) imply $\varphi(\ ) = f^{BR}(\ )$, completing the proof for Case 3, thus finalizing the proof of the theorem. □

Proof of Proposition 1 : Suppose that there is only one branch $b \in B$. Fixing the profile of cadet preferences $\in Q$, the baseline priority order p_b, and the BRADSO policy w_b^+, consider the

Proof of Lemma 1: Let s^* be a Nash equilibrium of the strategic-form game induced by the USMA-2020 mechanism $(S^{2020}, \varphi^{2020})$. Contrary to the claim suppose that, there exists $i, j \in I$ such that

$$\varphi_j^{2020}(s^*) \succ_i \varphi_i^{2020}(s^*) \quad \text{and} \quad i \; p_b \; j.$$

There are three possible cases, where in each case we reach a contradiction by showing that cadet i has a profitable deviation by mimicking the strategy of cadet j:

Case 1: $\varphi_j^{2020}(s^*) = (b, t^0)$ and $\varphi_i^{2020}(s^*) = (b, t^+)$.

Since by assumption $\varphi_i^{2020}(s^*) = (b, t^+)$,

$$s_i^* = b.$$

Moreover the assumptions $\varphi_j^{2020}(s^*) = (b, t^0)$, $\varphi_i^{2020}(s^*) \neq (b, t^0)$, and $i \; p_b \; j$ imply

$$j \in I^+(s^*) \quad \text{and} \quad s_j^* = \emptyset \tag{21}$$

But then, relation (21) and the assumption $i \; p_b \; j$ imply that, for the alternative strategy $\hat{s}_i = \emptyset$ for cadet i,

$$i \in I^+(s_{-i}^*, \hat{s}_i), \qquad (s^{fl})\text{cadet}$$

Since by assumption $\varphi_j^{2020}(s^*) = (b, t^+)$,

$$j \in I^+(s^*) \quad \text{and} \quad s^*_j = b. \tag{23}$$

Moreover, since $\varphi_i^{2020}(s^*) = \emptyset$ by assumption,

$$i \notin I^+(s^*).$$

Therefore, since $i \,\pi_b\, j$ by assumption,

$$j \in I^+(s^*) \text{ and } i \notin I^+(s^*) \quad \Longrightarrow \quad s^*_i = \emptyset.$$

But then, again thanks to assumption $i \,\pi_b\, j$, the relation (23) implies that, for the alternative strategy $\hat{s}_i = b$ for cadet i,

$$i \in I^+(s^*_{-i}, \hat{s}_i),$$

and thus

$$\underbrace{\varphi_i^{2020}(s^*_{-i}, \hat{s}_i)}_{\in \{(b,t^0),(b,t^+)\}} \succ_i \varphi_i^{2020}(s^*),$$

contradicting s^* is a Nash equilibrium strategy,[32] completing the proof for Case 3, and concluding the proof of Lemma 1. $\diamond$

For the next phase of our proof, we rely on the construction in the Step 2 of the mechanism φ^{BR}: Let I^0 be the set of q_b^0 highest π_b-priority cadets in I, and I^1 be the set of q_b^+ highest π_b-priority cadets in $I \setminus I^0$. Relabel the set of cadets in I^1, so that i^1 is the lowest π_b-priority cadet in I^1, i^2 is the second lowest π_b-priority cadet in I^1, $\ldots$, and $i^{q_b^+}$ is the highest π_b-priority cadet in I^1. Note that, cadet i^1 is the q^{th} highest π_b-priority cadet in set I, cadet i^2 is the $(q-1)^{th}$ highest π_b-priority cadet in set I, and so on. Let $J^0 = \{j \in I \setminus (I^0 \cup I^1) : (b, t^+) \succ_j \emptyset\}$. Assuming Step 2.n is the last sub-step of Step 2 of the mechanism φ^{BR}, for any $\ell \in \{1, \ldots, n\}$, let

$$J^\ell = \begin{cases} J^{\ell-1} & \text{if } \emptyset \succ_{i^\ell} (b, t^+) \\ J^{\ell-1} \cup \{i^\ell\} & \text{if } (b, t^+) \succ_{i^\ell} \emptyset \end{cases}$$

Recall that, under the mechanism φ^{BR}, exactly n cadets receive an assignment of (b, t^+). We will show that, the same is also the case under the Nash equilibria of the strategic-form game induced by the USMA-2020 mechanism $(S^{2020}, \varphi^{2020})$.

Let s^* be a Nash equilibrium of the strategic-form game induced by the USMA-2020 mechanism $(S^{2020}, \varphi^{2020})$. We have three cases to consider:

Case 1: $n = 0$

[32] Unlike the first two cases, in this case cadet i may even get a better assignment than cadet j (i.e. cadet i may receive an assignment of (b, t^0)) by mimicking cadet j's strategy.

Since by assumption $n = 0$ in this case,

$$j \in J^0 : (j, t^+) \; w_b^+ \; (i^1, t^0) = \emptyset \tag{24}$$

Towards a contradiction, suppose there exists a cadet $i \in I \setminus (I^0 \cup I^1)$ such that $i \in I^+(s)$. Since cadet i^1 is the q^{th} highest p_b-priority cadet in I, the assumption $i \in I^+(s)$ and relation (24) imply) that

$$i \notin J^0 \implies \emptyset \quad {}_i (b, t^+). \tag{25}$$

Moreover, since s that

Moreover, since cadet i is not one of the q highest p_b-priority cadets in I, ,

$$i \in I^+(s) \implies s_i = b. \tag{26}$$

But this means cadet i can instead submit an alternative strategy $\hat{s}_i = \emptyset$, assuring that she

1. $\varphi^{2020}_{i^\ell}(\sigma^*) = (b,t^+) \iff (b,t^+) \succ_{i^\ell} \emptyset$, and

2. $\varphi^{2020}_{i}(\sigma^*) = (b,t^+)$ for any $i \in \overline{J^\ell}$.

Proof of Lemma 2: Let σ^* be a Nash equilibrium of the strategic-form game induced by the USMA-2020 mechanism $(S^{2020}, \varphi^{2020})$. First recall that,

$$\text{for any } j \in I \setminus (I^0 \cup I^1), \qquad \varphi^{2020}_j(\sigma^*) \in \{(b,t^+), \emptyset\},$$

and therefore, since any cadet $j \in I \setminus (I^0 \cup I^1 \cup J^0)$ prefers remaining unmatched to receiving a position at the increased cost t^+ and she can assure remaining unmatched by submitting the strategy $s_j = \emptyset$,

$$\text{for any } j \in I \setminus (I^0 \cup I^1 \cup J^0), \qquad \varphi^{2020}_j(\sigma^*) = \emptyset \tag{30}$$

Also, by the mechanics of the Step 2 of the mechanism φ^{BR},

$$\text{for any } \ell \in \{1,\ldots,n\}, \qquad \{j \in J^{\ell-1} : (j,t^+)\ \omega^+_b\ (i^\ell, t^0)\} \neq \emptyset. \tag{31}$$

The proof of the lemma is by induction on ℓ. We first prove the result for $\ell = 1$.

Consider the highest π_b-priority cadet j in the set $\{j \in J^0 : (j,t^+)\ \omega^+_b\ (i^1, t^0)\}$. By relation 31, such a cadet exists.

First assume that $(b,t^+) \succ_{i^1} \emptyset$. In this case, $J^1 = J^0 \cup \{i^1\}$ and cadet i^1 is the highest π_b-priority cadet in J^1. Hence $\overline{J^1} = \{i^1\}$ in this case. Consider the Nash equilibrium strategies of cadet i^1 and cadet j. If $s_{i^1} = \emptyset$, then by relation (30) her competitor cadet j can secure himself an assignment of (b,t^+) by reporting a strategy of $s_j = b$, which would mean cadet i^1 has to remain unassigned, since by Lemma 1 no cadet in $I^0 \cup I^1$ can envy the assignment of cadet i^1 at Nash equilibria. In contrast, reporting a strategy of $s_{i^1} = b$ assures that cadet i^1 receives a position, which is preferred at any price to remaining unmatched by assumption $(b,t^+) \succ_{i^1} \emptyset$. Therefore, $s_{i^1} = b$, and hence

$$(b,t^+) \succ_{i^1} \emptyset \implies \begin{cases} \varphi^{2020}_{i^1}(\sigma^*) = (b,t^+), & \text{and} \\ \varphi^{2020}_{i}(\sigma^*) = (b,t^+) & \text{for any } i \in \overline{J^1} = \{i^1\}. \end{cases} \tag{32}$$

Next assume that $\emptyset \succ_{i^1} (b,t^+)$. In this case $J^1 = J^0$ and cadet j is the highest π_b-priority cadet in J^1. Hence $\overline{J^1} = \{j\}$ in this case. By Lemma 1, no cadet in $(I^0 \cup I^1) \setminus \{i^1\}$ can envy the assignment of cadet i^1 at Nash equilibria. Therefore, a strategy of $s_{i^1} \ni (b,t^+)$

Case 2. $n \in \{1, \ldots, q_b^+ - 1\}$

For this case, by the mechanics of the Step 2 of the mechanism φ^{BR},

$$\left|\{j \in J^n : (j, t^+) \; \omega_b^+ \; (i^{n+1}, t^0)\}\right| = n. \tag{37}$$

Consider cadet i^{n+1}. There are $q - (n+1)$ cadets with higher p_b-priority, and by relation (37) there are n cadets in J^n whose increased-cost assignments have higher ω_b^+ priority under the BRADSO policy than the base-cost assignment for cadet i^{n+1}. For any other cadet $i \in I \setminus J^n \cup I^0 \cup I^1 \setminus \{i^1, \ldots, i^{n+1}\}$ with $(i, t^+) \; \omega_b^+ \; (i^{n+1}, t^0)$, we must have $\emptyset \; _i \; (b, t^+)$ since $J^n \; J^0$. Therefore none of these individuals can receive an assignment of (b, t^+) under a Nash equilibrium strategy, and hence the number of cadets who can have higher $p_b^+(s)$-priority than cadet is i^{n+1} is at most $q - (n+1) + n = q - 1$ under any Nash equilibrium strategy. That is, cadet $i^{n+1} \in I^+(s)$ regardless of her submitted strategy, and therefore,

$$\varphi^{2020}_{i^{n+1}}(s) = (b, t^0), \tag{38}$$

since her best response

$\succ \in \mathcal{Q}^{|I|}$.

Individual rationality: No cadet $i \in I$ ever makes a proposal to a branch b at the increased cost t^+ under the cumulative offer process, unless her preferences are such that $(b,t^+) \succ_i \emptyset$. Hence the mechanism φ^{COM-BR} satisfies individual rationality.

Non-wastefulness: For any branch $b \in B$, unless there are already q contracts with distinct cadets on hold, it is not possible for all contracts of any given cadet to be rejected at any stage of the cumulative offer process under the choice rule C_b^{BR}. Hence the mechanism φ^{COM-BR} satisfies non-wastefulness.

Lack of priority reversals: Suppose that $\varphi_j^{COM-BR}(\succ) \succ_i \varphi_i^{COM-BR}(\succ)$ for a pair of cadets $i, j \in I$. Since the mechanism φ^{COM-BR} is individually rational, $\varphi_j^{COM-BR}(\succ) \neq \emptyset$. Let branch $b \in B$ and cost $t \in \{t^0, t^+\}$ be such that $\varphi_j^{COM-BR}(\succ) = (b,t)$. Let k be the final step of the cumulative offer process. Since $\varphi_j^{COM-BR}(\succ) \succ_i \varphi_i^{COM-BR}(\succ)$, cadet i has proposed the contract (i,b,t) to branch b at some step of the cumulative offer process, which is rejected by branch b (strictly speaking for the first time) either immediately or at a later step. Since the proposed contracts remain available until the termination of the procedure under the cumulative offer process, [34] the contract (i,b,t) is also rejected by branch b at the final Step k of the cumulative offer process. In contrast, since $\varphi_j^{COM-BR}(\succ) = (b,t)$, contract (j,b,t) is chosen by branch b at the final step k of the cumulative offer process. If the contract (j,b,t) is accepted as one of the first q_b^0 positions under the choice rule C_b^{BR}, then $(j,b,t) \succ_{w_b^0} (i,b,t)$. Otherwise, if the contract (j,b,t) is accepted as one of the last q_b^+ positions under the choice rule C_b^{BR}, then $(j,b,t) \succ_{w_b^+} (i,b,t)$. In either case we have $j \; p_b \; i$, proving that the mechanism φ^{COM-BR} has no priority reversals.

Enforcement of the BRADSO policy: First suppose that cadets $i, j \in I$ are such that $\varphi_i^{COM-BR}(\succ) = (b,t^+)$ and $(b,t^0) \succ_j \varphi_j^{COM-BR}(\succ)$. The relation $(b,t^0) \succ_j \varphi_j^{COM-BR}(\succ)$ implies that cadet j has proposed the contract (j,b,t^0) to the branch b at some step of the cumulative offer process, which is rejected by branch b either immediately or at a later step. Let k be the final step of the cumulative offer process. Since the proposed contracts remain available until the termination of the procedure under the cumulative offer process, the contract (j,b,t^0) is also rejected by branch b at the final Step k of the cumulative offer process. More specifically, it is rejected by the choice rule C_b^{BR} at the final Step k both for the first q_b^0 positions using the native priority order $\succ_{w_b^0}$ and for the last q_b^+ positions using the BRADSO policy $\succ_{w_b^+}$. In contrast, contract (i,b,t) is chosen by branch b at the final Step k of the cumulative offer process using the BRADSO policy $\succ_{w_b^+}$. Therefore,

$$\left.\begin{array}{l} \varphi_i^{COM-BR}(\succ) = (b,t^+), \text{ and} \\ (b,t^0) \succ_j \varphi_j^{COM-BR}(\succ) \end{array}\right\} \implies (i,t^+) \succ_{w_b^+} (j,t^0). \qquad (40)$$

Next suppose that cadets $i, j \in I$ are such that $\varphi_j^{COM-BR}(\succ) = (b,t^0)$, $(b,t^+) \succ_i \varphi_i^{COM-BR}(\succ)$, $(i,t^+) \succ_{w_b^+} (j,t^0)$, and moreover, let cadet j be the lowest p_b-priority cadet with an assignment of $\varphi_j^{COM-BR}(\succ) = (b,t^0)$. The relation $(b,t^+) \succeq_i \varphi_i^{COM-BR}(\succ)$ implies that cadet i has proposed the

[34] It is this feature of the cumulative offer process that is emphasized in its name.

contract (j, b, t^+) to the branch b at some step of the cumulative offer process, which is rejected by branch b either immediately or at a later step. Let k be the final step of the cumulative offer process. Since the proposed contracts remain available until the termination of the procedure under the cumulative offer process, the contract (j, b, t^+) is also rejected by branch b at the final Step k of the cumulative offer process. More specifically, it is rejected by the choice rule C_b^{BR} at the final Step k even for the last q_b^+ positions using the BRADSO policy ω_b^+. Therefore, since by assumption we have $(i, t^+)\ \omega_b^+\ (j, t^0)$, cadet j must have received one of the first q^0 positions using the native priority order ω_b^0. But since cadet j is the lowest π_b-priority cadet with an assignment of $\varphi_j^{COM \oplus BR}(\succ) = (b, t^0)$, that means no cadet has received any of the last q_b^+ positions at the base cost of t^0. Therefore, since $\varphi^{COM \oplus BR}$ satisfies non-wastefulness,

$$\left.\begin{array}{l} \varphi_j^{COM \oplus BR}(\succ) = (b, t^0), \\ (b, t^+) \succ_i \varphi_i^{COM \oplus BR}(\succ), \text{ and} \\ (i, t^+)\ \omega_b^+\ (j, t^0) \end{array}\right\} \implies \left|\{i' \in I : \varphi_{i'}^{COM \oplus BR}(\succ) = (b, t^+)\}\right| = q_b^+. \tag{41}$$

Relations (40) and (41) imply that mechanism $\varphi^{COM \oplus BR}$ satisfies enforcement of the BRADSO policy.

Strategy-proofness: Our model is a special case of matching problems with slot-specific priorities by Kominers and Sönmez (2016). Hence strategy-poofness of the mechanism $\varphi^{COM \oplus BR}$ is a direct corollary of their Theorem 3, which proves strategy-proofness of the cumulative offer mechanism more broadly for matching problems with slot-specific priorities.

Uniqueness: We prove uniqueness via two lemmata.

Lemma 3. Let $X, Y \in \mathcal{A}$ be two distinct allocations that satisfy individual rationality, non-wastefulness, enforcement of BRADSO policy, and have no priority reversals. Then there exists a cadet $i \in I$ who receives non-empty and distinct assignments under X and Y.

Proof of Lemma 3: The proof is by contradiction. Fix $\succ \in \mathcal{Q}^{|I|}$. Let $X, Y \in \mathcal{A}$ be two distinct allocations that satisfy individual rationality, non-wastefulness, enforcement of BRADSO policy, and have no priority reversals. To derive the desired contradiction, suppose that, for any cadet $i \in I$,

$$X_i \neq Y_i \implies X_i = \emptyset \text{ or } Y_i = \emptyset \tag{42}$$

Pick any branch $b \in B$ such that $X_b \neq Y_b$. Let $j \in I$ be the highest π_b-priority cadet who is assigned to branch b either under X or under Y but not both. W.l.o.g., let cadet j be assigned to branch b under allocation X but not under allocation Y. By relation (42),

$$Y_j = \emptyset.$$

Since allocation Y satisfies non-wastefulness, there exists a cadet $k \in I$ who is assigned to branch b

under allocation Y but not under allocation X. By relation (42),

$$X_k = Æ,$$

and therefore, by choice of cadet j, cadet k has lower p_b-priority than cadet j. Moreover, since allocation Y has no priority reversals and $Y_j = Æ$, we have

$$Y_k = (b, t^+), \tag{43}$$

and since allocation Y satis es (condition 1 of) the axiom enforcement of BRADSO policy, we have

$$(k, t^+) \; w_b^+ \; (j, t^0). \tag{44}$$

Also relation (43) and individual rationality allocation Y imply

$$(b, t^+) \quad {}_k \; Æ. \tag{45}$$

De ne

$$I \quad f \; i \; 2 \; I : X_i = (b, t^+) g.$$

=8a25[(,)]TJ/F131 10.9091 Tf 4.682 0 Td [(t)]TJ/F149 8.3049 Tf 3.868 4.505 Td [(+)]TJ/F149 11.3673 Tf 7.19 -4.505

Since allocation

k

Y

Proof of Lemma 4: The proof of the lemma is inspired by a technique introduced by Hirata and Kasuya (2017). Towards a contradiction, suppose there exists two distinct direct mechanisms φ and ψ that satisfy individual rationality, non-wastefulness, enforcement of BRADSO policy, strategy-proofness, and have no priority reversals. Let the preference profile $\succ \in Q^{|I|}$ be such that,

1. $\varphi(\succ) \neq \psi(\succ)$, and
2. the aggregate number of acceptable contracts between all cadets is minimized among all preference profiles $\tilde{\succ} \in Q^{|I|}$ such that $\varphi(\tilde{\succ}) \neq \psi(\tilde{\succ})$.

Let $X = \varphi(\succ)$ and $Y = \psi(\succ)$. By Lemma 3, there exists a cadet $i \in I$ such that

1. $X_i \neq \emptyset$,
2. $Y_i \neq \emptyset$, and
3. $X_i \neq Y_i$.

Since both allocations X and Y satisfy individual rationality,

$$X_i \succ_i \emptyset \quad \text{and} \quad Y_i \succ_i \emptyset$$

W.l.o.g., assume

$$X_i \succ_i Y_i \succ_i \emptyset$$

Construct the preference relation $\succ'_i \in Q$ as follows:

If $X_i = (b, t^0)$ for some $b \in B$, then

$$(b, t^0) \succ'_i \emptyset \succ'_i (b', t') \qquad \text{for any } (b', t') \in B \times T \setminus \{(b, t^0)\}.$$

Otherwise, if $X_i = (b, t^+)$ for some $b \in B$, then

$$(b, t^0) \succ'_i (b, t^+) \succ'_i \emptyset \succ'_i (b', t') \qquad \text{for any } (b', t') \in B \times T \setminus \{(b, t^0), (b, t^+)\}.$$

Since $X_i \succ_i Y_i \succ_i \emptyset$ and $(b, t^0) \succ_i (b, t^+)$, the preference relation $\succ'_i$ has strictly fewer acceptable contracts for cadet i than the preference relation $\succ_i$.

By strategy-proofness of the mechanism ψ, we have

$$\underbrace{\psi_i(\succ_i, \succ_{-i})}_{= Y_i} \succeq_i \psi_i(\succ'_i, \succ_{-i}),$$

and since no branch-cost pair $(b', t') \in B \times T$ with $Y_i \succ'_i (b', t')$ is acceptable under $\succ'_i$, by individual rationality of the mechanism ψ we have

$$\psi_i(\succ'_i, \succ_{-i}) = \emptyset \tag{49}$$

Similarly, by strategy-proofness of the mechanism j , we have

$$j_{i}(\ {}^{0}_{i},\ \ _{i})\ \ {}^{0}_{i}\ \underbrace{j_{i}(\ _{i},\ \ _{i})}_{=\,x_{i}},$$

which in turn implies

$$j_{i}(\ {}^{0}_{i},\ \ _{i}) \neq Æ. \tag{50}$$

But then, by relations (49) and (50) we have

$$j(\ {}^{0}_{i},\ \ _{i}) \neq y(\ {}^{0}_{i},\ \ _{i}),$$

giving us the desired contradiction, since between all cadets the preference pro le $(\ {}^{0}_{i},\ \ _{i})$ has strictly fewer acceptable contracts than the preference pro le . This completes the proof of Lemma 4. }

Table 1: Branches and Applications for Classes of 2020 and 2021

Figure 2: BRADSO Ranking Relative to Non-BRADSO Ranking by Class of 2021

Notes. This gure reports where in the preference list a branch is ranked with BRADSO relative to where it is ranked without BRADSO. A value of 1 (2 or 3) indicates that the branch is ranked with BRADSO immediately after (two places or three places after, respectively) the branch is ranked at base cost. 4+ means that the a branch is ranked with BRADSO four or more choices after the branch is ranked at base cost.

Figure 3: USMA-2006 and USMA-2020 Mechanism Performance under Truthful Strategies Simulated from Preference Data from Class of 2021

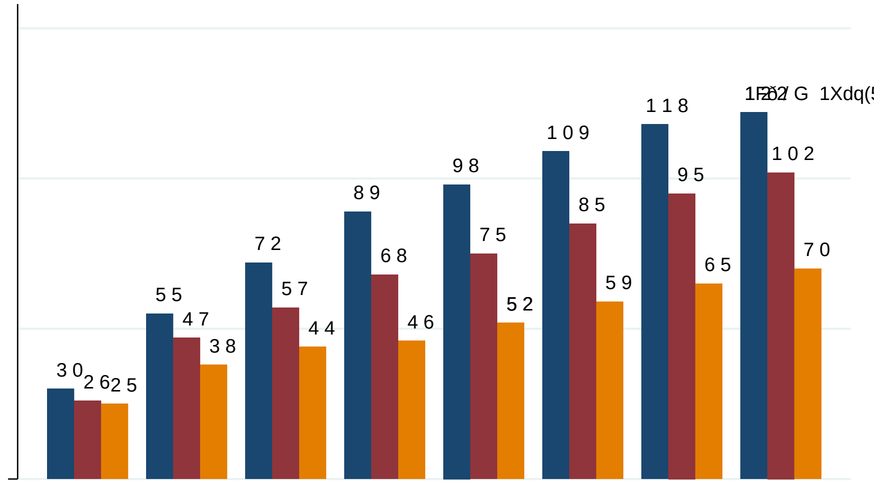
Notes. USMA used the strategy-proof COM-BRADSO mechanism for the Class of 2021. This gure uses data from the Class of 2021 on cadet preferences, branch priorities, and branch capacities to simulate the outcomes of the mechanisms USMA-2006 and USMA-2020. Since the strategy spaces of the mechanisms USMA-2006 and USMA-2020 differ from that of the mechanism COM-BRADSO, cadet strategies that correspond to truthful branch-preferences and BRADSO willingness are are simulated from cadet preferences over branch-cost pairs under the COM-BRADSO mechanism. Truthful strategies for the mechanisms USMA-2006 and USMA-2020 are constructed from Class of 2021 preferences by assuming that a preference indicating willingness to BRADSO at a branch means the cadet's strategy under the USMA-2006 a N2sindica adica
fhre a N2sin47wDetectablesin47wPioritiysin47wRe

Figure 4: USMA-2020 Mechanism Performance Under Indicative and Final Strategies

Notes. This gure reports on the number of Strategic BRADSOs, BRADSO-IC failures, Detectable Priority Reversals, and Priority Reversals under indicative strategies submitted in a dry-run of the USMA-2020 mechanism and nal strategies of the USMA-2020 mechanism for the Class of 2020.

Figure 5: USMA-2020 Mechanism Performance under Truthful Strategies Simulated from Indicative and Final Preference Data from Class of 2021

Figure 6: Number of BRADSOs Charged Across BRADSO Policies and Cap Sizes



Notes. This gure reports on the number of BRADSOs charged for three BRADSO policies: Ultimate BRADSO, BRADSO-2020, and BRADSO-2021 using data from the Class of 2021. The BRADSO cap ranges from 5% to 75% of slots at each branch. Each outcome is computed by running COM-BRADSO given stated cadet preferences under different BRADSO policies and cap sizes.

B Online Appendix: Supplementary Material

B.1 Individual-Proposing Deferred Acceptance

The USMA-2020 mechanism was based on the individual-proposing deferred acceptance algorithm (Gale and Shapley, 1962). Given a ranking over branches, the individual-proposing deferred acceptance algorithm (DA) produces a matching as follows.

> Individual-Proposing Deferred Acceptance Algorithm (DA)
>
> Step 1: Each cadet applies to her most preferred branch. Each branch b tentatively assigns applicants with the highest priority until all cadets are chosen or all q_b slots as assigned and permanently rejects the rest. If there are no rejections, then stop.
>
> Step k: Each cadet who was rejected in Step k-1 applies to her next preferred branch, if such a branch exists. Branch b tentatively assigns cadets with the highest priority until all all cadets are chosen or all q_b slots are assigned and permanently rejects the rest. If there are no rejections, then stop.
>
> The algorithm terminates when there are no rejections, at which point all tentative assignments are nalized.

B.2 Cadet Survey Questions and Answers

In fall 2020, the Army administered a survey of cadets. This survey asked two questions related to assignment mechanisms, one on cadet understanding of USMA-2020 and the other on cadet preferences over assignment mechanisms. This section reports the questions and the distribution of survey responses.

Question 1. What response below best describes your understanding of the impact of volunteering to BRADSO for a branch in this year's branching process?

A. I am more likely to receive the branch, but I am only charged a BRADSO if I would have failed to receive the branch had I not volunteered to BRADSO. (43.3% of respondents)

B. I am charged a BRADSO if I receive the branch, regardless of whether volunteering to BRADSO helped me receive the branch or not. (9.5% of respondents)

C. I am more likely to receive the branch, but I may not be charged a BRADSO if many cadets who receive the same branch not only rank below me but also volunteer to BRADSO. (38.8% of respondents)

D. I am more likely to receive the branch, but I do not know how the Army determines who is charged a BRADSO. (6.7% of respondents)

E. I am NOT more likely to receive the branch even though I volunteered to BRADSO. (1.8 percent of respondents)

38.8% of cadets answered the correct answer (answer C). 43.3% of cadets believed that the 2020 mechanism would only charge a BRADSO if required to receive the branch (answer A)

Question 2. A cadet who is charged a BRADSO is required to serve an additional 3 years on Active Duty. Under the current mechanism, cadets must rank order all 17 branches and indicate if they are willing to BRADSO for each branch choice. For ex16(the)-215(corr)18(eple:O.)-363((1f23a.willi.l)-29i0s2(they4-5corr)18(eple:O.)9

Table B.1: Mechanism Replication Rate

			Percent Correct	
	Total Applicants	Number Incorrect	Branch	BRADSO
Applicant Class	(1)	(2)	(3)	(4)
2014	1006	28	97.2%	98.1%
2015	976	4	99.6%	100.0%
2016	951	11	98.8%	99.6%
2017	944	2	99.8%	100.0%
2018	963	11	98.9%	99.6%
2019	931	4	99.6%	100.0%
2020	1089	0	100.0%	100.0%
2021	994	0	100.0%	100.0%
All	7854	60	99.2%	99.7%

Notes. This table reports the replication rate of the USMA assignment mechanism across years. The USMA-2006 mechanism is used for the Classes of 2014-2019, USMA-2020 mechanism is used for the Class of 2020, and the COM-BRADSO mechanism is used for the Class of of 2021. Number incorrect are the number of cadets who obtain a different assignment under our replication. Branch percent correct is the number of branch assignments that we replicate. BRADSO percent correct is the number of BRADSO assignments we replicate.