on patients' decisions. In this paper, we consider an arbitrarily sized group of matched incompatible patient-donor pairs for whom the only feasible exchange of kidneys is a cyclic exchange. Because the patients' health statuses are dynamic, and transplantation surgeries take place simultaneously, we model the patients' transplant timing decisions as a noncooperative stochastic game. In Section 2, we present the stochastic game formulation for which we present our equilibrium analyses in Section 3. In Section 4, we gain insights by illustrating our model with real clinical data for a large, nationally representative cohort. Finally, in Section 5 we conclude the paper by highlights.

2. Model Formulation

We consider $N \geq 2$ self-interested patient-donor pairs where Patient i is compatible for an exchange with Donor $i+1$ for i

days or weeks) accrued in state $s \in S$ given an exchange does not occur. We also define $u_i(s, 1)$ as the *expected post-transplant reward* (e.g. expected quality-adjusted post-transplant survival) of Patient $i \in N$ given an exchange occurs in $s \in S$. Note that for each patient $i \in N$ and state $s \in S$, $u_i(s, 1)$ is a one-time lump-sum reward, where for each $i \in N$, $u_i(s, 1) = 0$ for all $s \in D$ as there is no possibility for an exchange in such states for the surviving patients. We assume that each patient i

her strategy unilaterally. Therefore, strategy profiles satisfying (2) are also known as *stationary-perfect equilibria* in the economics literature. In the remainder of the paper, unless otherwise stated, we let the terms “strategy” and “equilibrium” refer to “stationary strategy” and “stationary-perfect equilibrium”, respectively.

Recall that an exchange can occur in state $s \in S$ only if all patients choose a positive probability to offer to exchange. Therefore, a single patient can not affect the outcome as long as one of the other patients chooses to wait. As an intuitive consequence, Theorem 1 provides necessary and sufficient conditions for a strategy profile to be an equilibrium of game G.

Theorem 1 : *A strategy profile* $\mathbf{A}$ *is an equilibrium of game* G *if and only if for all* $s \in S$ *and* $i \in N$*:*

$$g_i(s, \mathbf{a}_i, \mathbf{a}_{-i}) = \max \Bigg\{ F_i\Big(s, \mathbf{g}_i(\mathbf{a}_i, \mathbf{a}_{-i})\Big),$$
$$u_i(s, 1) \prod_{j \in \mathcal{N}_{-i}} a_j(s) + \Bigg(1 - \prod_{j \in \mathcal{N}_{-i}} a_j(s)\Bigg) F_i\Big(s, \mathbf{g}_i(\mathbf{a}_i, \mathbf{a}_{-i})\Big)\Bigg\}. \qquad (3)$$

Because Nash equilibria are immune only to unilateral deviations, game G may admit a large number of pathological equilibria that make little clinical sense. For instance, by Theorem 1, any strategy profile under which at least two arbitrary, but not necessarily the same, patients offer to exchange with probability 0 in every state $s \in S$ denotes an equilibrium of game G. Furthermore, as different equilibria may imply different payoff outcomes, due to vast multiplicity of equilibria that game G can admit, a complete characterization of such equilibria is computationally prohibitive. As such, we consider equilibrium selection and motivate the following question: *Given the game starts in state* $\bar{s} \in S$*, which equilibrium maximizes the social welfare, i.e., the sum of the patients' total expected payo* an alternative equilibrium with an equivalent payoff profile in which there is no more than a single patient who randomizes in any state $\mathbf{s} \in S$, which we formally state in Lemma 1 (iii). For a given strategy profile A, for $\mathbf{s} \in S$, let $Y_{\mathbf{s}}(\mathrm{A}) = \{i \in N \,|\, a_i(\mathbf{s}) \in (0,1)\}$ denote the set of patie:he aine de a

Patient $j \in N$ with $\bar{y}_j(s) = 0$ randomizes between waiting and offering to exchange. The variable $\bar{z}$ enforces logical relationships among the $\bar{y}_i$ variables for $i \in N$.

Theorem 2 reveals the relationship between equilibria of game G and feasible solutions to Λ. For any equilibrium $\bar{A}$ of game G, one can construct a feasible solution $(\bar{w}, \bar{y}, \bar{z})$ to Λ from $\bar{A}$ in which $\bar{w}$ represents the payoff profile of $\bar{A}$. As a special case, when there are two patient-donor pairs, for any pair $(\bar{w}, \bar{y}, \bar{z})$ satisfying (4a)-(4j) one can construct an equilibrium $\bar{A}$ of game G from $\bar{w}$ and $\bar{y}$ with a payoff profile equivalent to $\bar{w}$. Note that for a feasible solution $(\bar{w}, \bar{y}, \bar{z})$ to Λ, while $\bar{w}$ represents the payoff profile of the equilibrium that $(\bar{w}, \bar{y}, \bar{z})$ induces, as $\bar{y}$ involve only binary valu25(P)27.6(1q0.l)0.2(u).4

We can interpret (5) as follows. In a pure equilibrium of game G, for Patient i there are two possible scenarios in each state $s \in S$: If some of the other patients want to wait, then because the decision of Patient i will not affect the occurrence of the exchange, she is indifferent between waiting and offering to exchange. Otherwise, she offers as well only if she benefits from the exchange.

4. Numerical Study

We illustrate our model using clinical data. As most of the social benefit is accrued by exchanges with three or fewer patients [38], we restrict our focus to two- and three-way exchanges. For convenience and consistency on notation, we present results only from two-way exchanges and describe results on three-way exchanges in the appendix. While maximizing the social objective, we estimate the cost of restricting our attention to pure equilibria, rather than randomized equilibria. After demonstrating that this cost appears to be negligible, we consider pure equilibria for the rest of the experiments.

4.1 Data Sources and Parameter Estimation

In this section, we estimate the transition probabilities and post-transplant rewards based on clinical data. There is a broad consensus among clinicians that glomerular filtration rate (GFR) is the best measure of remaining pre-dialysis kidney functionality for ESRD patients. Although the stages of ESRD are mainly based on measured or estimated GFR [28], it appears that no stochastic model of pre-dialysis GFR progression has been described in the literature. We use GFR levels and the patient's dialysis status to represent her health. To build a Markovian progression of pre-dialysis GFR levels, we use a data set from The Thomas E. Starzl Transplantation Institute at the University of Pittsburgh Medical Center (UPMC), one of the largest transplantation centers nationwide. This set provides detailed data on laboratory measurements for more than 60,000 ESRD patients, but due to

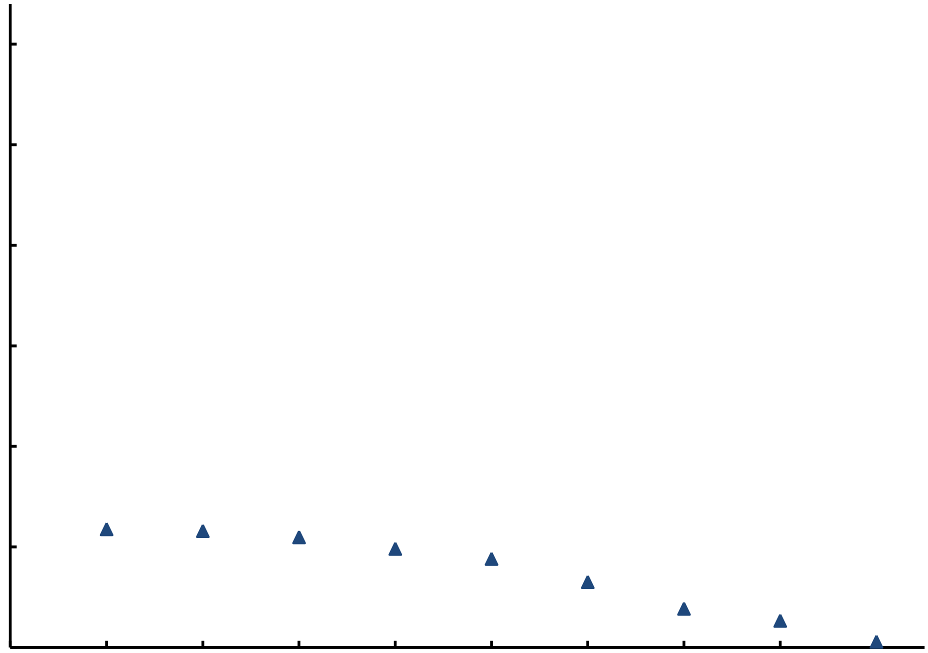

Figure 3: Social welfare loss and patients' individual welfare losses due to patient autonomy. The number next to each data label indicates the loss in absolute terms (in quality-adjusted life weeks).

benefits from the central decision-maker's decisions, and the impact of patient autonomy on her welfare is more dramatic in absolute and relative terms.

As the society's interest may conflict with patients' self-interests, a socially optimal equilibrium strategy may not be an optimal equilibrium strategy that a patient can play. Therefore, for each individual patient we calculate the cost of playing the socially optimal equilibrium strategy rather than any other equilibrium strategy. We let ${}^{i}A^{*} = ({}^{i}a_i^{*}, {}^{i}a_{-i}^{*})$ denote a pure equilibrium that maximizes Patient i

5. Highlights

We model the patients' transplant timing decisions in a cyclic PKE as a non-zero-sum stochastic game and analyze the resulting equilibrium selection problem from a social point of view. Our

Also by constraints (4e), (4f) and (4i) for any $i \in N$, $u_{-i}(s, 1) \geq w_{-i}(s)$ for all $s \in R_i$. By (16a) and the definition of Z_i for $i \in N$, this implies:

$$\text{For any } i \in N: \; u_{-i}(s, 1) > F_{-i}(s, \bar{w}_{-i}) \text{ for all } s \in R_i \setminus Z_i. \tag{16b}$$

Thus, by (16a) and (16b), for any $i \in N$, $\left(\frac{\bar{w}_{-i}(s) - F_{-i}(s, \bar{w}_{-i})}{u_{-i}(s, 1) - F_{-i}(s, \bar{w}_{-i})} \right) \in [0, 1]$ for all $s \in R_i \setminus Z_i$. Now, consider the strategy profile $\bar{A}$ defined by:

$$\bar{a}_i(s) =$$

3. If $s \in Z_2$, then since $u_1(s, 1) = F_1(s, \bar{w}_1)$, (23

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