# Designing Practical and Fair Sequential Team Contests[*]

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Abstract

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# 1 Introduction

In this paper, we model shootouts that are used as tiebreak mechanisms in several team sports such as football and hockey as a mechanism design problem with order independence of the outcome in mind: which team starts kicking first should not matter for the outcome of the shootout but lead to even chances of winning when all players are equally talented.

Economists have long recognized that the effect of the order of actions in sequential contests on performance of the contestants is far from negligible. Examples in specific sequential individual and team contests are plenty, e.g., R&D races (Fudenberg et al., 1983 and Harris and Vickers, 1985, 1987), dynamic games in general (Cabral, 2002), job promotions (Rosen, 1986), political campaigns (Klumpp and Polborn, 2006), music competitions (Ginsburgh and van Ours, 2003) as well as penalty shootouts in football matches (Apesteguia and Palacios-Huerta, 2010) and tennis matches (Cohen-Zada, Krumer, and Shapir, 2018 and the references therein).[1] Clearly, an order of actions that provides a systematic first- or second-mover advantage to one of the parties may decrease the probability of the `better' contestant to win, causing efficiency and fairness issues. Therefore, a focal direction is to aim for order independence in such team contests.

The history and experience of football and other sports' tiebreak mechanisms, known as penalty shootouts, present us a unique natural experiment to understand the strategic role of move order. Penalty shootouts currently constitute the only way to determine the winning team when the score is tied in major football elimination tournament matches after the regular 90-minute period and the 30-minute extra time, known as *overtime*. It is customary to use tiebreak mechanisms in many other sports as well to determine the eventual winner when the regular match ends with a tie, e.g., tennis, ice hockey, field hockey, water polo, handball, cricket, and rugby.

In a football shootout, since 1970 each team takes five penalty kicks from the penalty mark in fixed order (ABAB for short, meaning that Team A kicks first then Team B kicks, then in the second round Team A kicks first again and so on), and the order of the kicks has always been decided by the referee's initial even coin toss. If the shootout score is tied after each team takes five penalty kicks, sudden-death rounds are reached, which go on until the tie is broken, such that the kicking order remains the same as regular rounds.[2]

A particular observation shared by multiple empirical studies regarding football penalty shootouts is that the degree of how much the kicking order in the ABAB mechanism matters may differ across different football competitions/traditions. For example, although kicking order does not matter for the German national cup, the Spanish national cup shootouts favor first-kicking teams significantly. On the other hand, in English cups, the first-kicking team has only a slight advantage.

cal outcomes in terms of first- and second-moving teams' winning chances. In addition, some researchers provide evidence that the first-kicking team wins *significantly* more often overall with ABAB, while some others dispute some of that evidence.[4] No study, however, provides evidence that the second-kicking team wins more often overall.

Shootouts tend to be shorter and more structured than a regular match. They can be modeled like dynamic versions of contests. We introduce such a model in which the kickers not only care about their team's winning the shootout but also about the individual performance they display during taking their penalty shot. We provide empirical evidence to support this modeling assumption from Bar-Eli and Azar (2009) and Almeida, Volossovitch, and Duarte (2016): even high level players often aim at safer spots where the kick can be saved more often by the goalie than optimal spots, which provide higher chances of scoring, but also higher chances of kicking out. To capture this feature of penalty kicks, we assume that for a kicker, a save of his kick by a goalie is less irritating and more desirable than kicking the penalty out (as in the former case, the miss is caused by somebody else's, i.e., goalkeeper's, luck or effort, but not by the kicker's own mistake as in the latter case). We explain this empirical evidence in detail in Section 2.

Then we define *order independence* as the requirement that equally balanced teams – in terms of their players' shootout abilities – have equal chance of winning any time when the score is tied at the beginning of any round, i.e., after equal numbers of attempts, under all state-symmetric equilibria of the induced shootout game.[5,6] Note that this property has implications only when the score is tied at the beginning of a round but is silent when it is not tied. Thus, it implies ex-ante fairness, i.e., an equal chance of winning at the beginning of the shootout at all state-symmetric equilibria even following a totally unfair coin toss.

First, we characterize order-independent mechanisms in regular rounds in Theorem 1. All exogenous mechanisms that have a predetermined kicking-order pattern – with one exception – are found to be order dependent, even if the sudden-death rounds were order independent for these mechanisms, e.g., even if the winner were determined by an even coin flip in sudden death. There is only one class of *exogenous* order-independent mechanisms, in which the kicking order after Round 1 is determined by an even coin flip in each round.

An important implication of this finding is that, as ABAB and ABBA have exogenous orders, regardless of the initial coin flip to determine which team goes first, they are both order dependent in regular rounds.

The whole class of order-independent mechanisms in regular rounds has the feature that when

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[4] See Apesteguia and Palacios-Huerta (2010) for evidence on the first kicking team winning *significantly* more often. Kocher, Lenz, and Sutter (2012), on the other hand, dispute this finding. Later Palacios-Huerta (2014) uses a larger data set to find again a first-mover advantage (see also Figure A.1 in Appendix D.1).

[5] A precursor of our concept of order independence can be found in Che and Hendershott (2008), who use it for only one round in which teams take turns.

[6] A state-symmetric equilibrium is a Markov perfect Bayesian equilibrium in which each kicker uses the same strategy when the state of the game defined by the score difference and kicking order is (symmetrically for each team) the same at the round he moves.

the score is tied at the beginning of a round, the kicking order does not matter for that round. Thus, we obtain order independence at the beginning of the shootouts.

On the other hand, all other order-independent mechanisms have adgen kicking-order determination when score is not tied at the beginning of a round: the probability of which team moves rst in this round is the same for both teams whenever they are in each otheshes . E.g., consider two cases in which Team 1 is ahead 10 and Team 2 is ahead 0 1 at the beginning of Round 2, respectively; then Team 1's probability of moving rst in Round 2 in the rst case is the same as Team 2's probability of moving rst in the second case. We refer to the class of mechanisms that fully characterize the continuum of order-independent mechanisms in regular rounds aesn seo ymtic .

Then we move from regular rounds to sudden-death rounds. That is, we consider order independence in sudden-death rounds when the score is tied after regular rounds. Interestingly, as the score is never uneven at the beginning of any sudden-death round, both ABAB and ABBA are tautologically uneven score symmetric in sudden-death rounds. ABAB induces an in nite game such that each sudden-death round is a repetition of the previous one and the game only ends when one team scores and the other does not. We show that ABAB is not order-independent in sudden-death rounds for reasons very di erent than those for regular rounds (Theorem 2). It turns out that ABAB leads to multiplicity of equilibria as in that game: For every equilibrium in which Team 1 wins more often, there is a dual equilibrium in which Team 1 and Team 2 players swap their strategies, and hence Team 2 wins more often, and yet there is always an equilibrium in which both teams win with equal probability.[7]

On the other hand, we show in Theorem 3 that alternating order of the teams as in ABBA is enough to rule out asymmetric equilibria in which teams win with di erent probabilities as state-symmetric and to sustain order independence back. Then we provide a large class of order-independent mechanisms in sudden-death rounds in Theorem 4.

for the two dimensional goal for tractability purposes to capture these nuances in revealed kicker utility functions.

When we analyze our kicker optimal strategy, the resulting behavior mimics the empirical findings using our utility function representation: kickers end up aiming at a safer spot instead of goal-optimal spot so that they can avoid the higher likelihood outcome of kicking out. Therefore, our utility function provides a rational explanation for this revealed kicker behavior.[13]

Besides this compelling evidence regarding penalty kick performance of kickers relying how a goal is missed, the relevant literature also points out that overall players care about their own performance, besides their team's outcome in other dynamic team contests. Chapsal and Vilain (2019) provide evidence from international team squash tournaments that players care not only about their team's win or loss, but also their individual performance.

# 3 Model

## 3.1 The Setup

Two football teams, which we refer to as Team 1 ($T_1$ in mathematical notation) and Team 2 ($T_2$ in mathematical notation), are facing off in a penalty shootout. Each team shall take $n$ sequential rounds of penalty shots. Each round consists of one team kicking first, and, after observing the outcome of that shot, the second team taking the next shot. If one team scores more goals than the other at the end of $n$ rounds, then it wins the match. We refer to these $n$ rounds as the *regular rounds*. Throughout the paper we will assume that $n = 2$. This is sufficient to characterize order independence and analyze the current scheme, ABAB, as well as other proposed mechanisms, such as the alternating-order mechanism, ABBA. Thus, with $n = 2$, the analysis is tractable and yet rich enough to capture the multi-round feature of penalty shootouts.[14]

decision because Taffarel did go to his left, and he would never have got to the shot I planned. Unfortunately, and I don't know how, the ball went up three meters and flew over the crossbar. I failed that time. Period. And it affected me for years. It is the worst moment of my career. I still dream about it. If I could erase a moment from my career, it would be that one."

[13] We also infer from Baggio's quote in Footnote 12 that goalies typically feel the need to dive at the time the ball is kicked. This is because, at the optimal speed-accuracy combinations of world-class kickers, the kicked ball typically takes around 0.3 seconds to reach the goal line (see, e.g., Harford, 2006, Chiappori et al., 2002, and Palacios-Huerta, 2003), which is less than the total of (1) roughly 0.2 seconds' reaction time of the goalie to clearly recognize the kick direction of the ball first, plus (2) the time during his dive to reach the expected arrival spot of the ball before it reaches the goal plane. Hence, a goalie cannot afford to wait until he clearly observes the kick direction: to prevent a goal with non-trivial probability, he must commit to pick a side to dive – or alternatively to stay in the middle. As Baggio's quote also indicates, a shot aimed at the middle may be missed outright or may hit the feet or the legs of the diving goalie that cover part of the middle; thus, the shot can be saved even if the goalie dives.

[14] We have $n = 3$ results in Appendix G, and no extra insight are obtained in this analysis. Similarly, we skip $n > 3$ as the analysis becomes extremely cumbersome and lengthy. Although, we do not have a proof for $n > 3$, we have no reason to suspect it would not generalize to this setting.

If the shootout score is tied at the end of n regular rounds, the format reverts to sudden death ;

Although so far we developed our theory taking football as our primary application, the insights we discover apply to other contests and sports. In particular, we can classify penalty shootouts as easy-task or difficult-task based on the goal scoring probability $P_G(x)$. A shootout is easy if $P_G(x) > \frac{1}{2}$ for all $x \in [0; \bar{x}]$. A shootout is difficult if $P_G(x) < \frac{1}{2}$ for all $x \in [0; \bar{x}]$. A football shootout is an example of an easy task, while a hockey shootout is an example of a difficult task. [18] This distinction will not matter in our results until we discuss different efficiency notions and practical design considerations in Appendix F. We assume that the shootout is either easy or difficult, but not mixed, throughout the paper. Thus, our analysis will focus on these two cases throughout.

Function $P_O$, on the other hand, is an increasing twice continuously differentiable convex function. Increasing $P_O$ is straightforward to motivate: the closer to the middle the ball is aimed, the lower is the chance that the ball will go out. Single-peakedness of $P_G$ is also easy to motivate: Whenever the ball is aimed at low $x$ values, it can be saved with a higher chance by the diving goalie (see Footnote 18 for hockey dynamics). For higher $x$ values, although the goalie's chances of saving the ball decrease as he may no longer be able to reach it, the chances of the ball going out increase. Hence, it is easy to motivate the unique spot $\bar{x}$, which maximizes the goal probability. We will refer to it as the goal-optimal spot. Concavity of $P_G$ and convexity of $P_O$ are primarily assumed for the tractability of our analysis, and do not play any other major role for the interpretation of our results.

We assume that each kicker on both teams is identical in ability and has the same goal-scoring and kicking-out probability.[19]

## 3.2 Shootout Mechanisms and the Shootout Game

A shootout mechanism is a function, , that assigns a probability $(h^{k-1}; g_{T_1} : g_{T_2})$ to Team 1 kicking first in Round $k$, given the sequence of first-kicking teams in the first $k-1$ rounds is $h^{k-1} = (h_r^{k-1})_{r=1}^{k-1}$ where $h_r^{k-1} \in \{T_1; T_2\}$ is the team that kicked first in Round $r$ and $g_{T_1} : g_{T_g}$

kicking teams in the previous $k \quad 1$ rounds $h^{k \quad 1}$, and feasible scores $g_{T_1} : g_{T_2}$, the Nature determines with probability $\quad (h^{k \quad 1}; g_{T_1} : g_{T_2})$ Team 1 kicking next  rst and probability $1 \quad\quad (h^{k \quad 1}; g_{T_1} : g_{T_2})$ Team 2 kicking next  rst. Then a kicker of the  rst-kicking team takes the penalty shot, observing the state and the history of the outcomes of all the shots up to that point as goal, out, or save. The kicker aims at his intended spot $x$ 2 $[0; 1]$ to maximize his expected individual payo  (which we explain in the next paragraph). Then the Nature determines with probability distribution $P_G(x); P_O(x); 1 \quad P_G(x) \quad P_O(x)$ whether the penalty kick results in a goal, goes out, or is saved, respectively. After the outcome of this shot is observed, the other team's kicker takes a penalty shot, observing the history of the outcomes of the shots up to that point. We continue until the end of regular rounds, Round $k = n$, similarly. If the score is tied after the last regular round, sudden-death rounds take place until the tie is broken at the end of a sudden-death Round $k > n$.

Each kicker aims to maximize his expectedtended about others' kicks are only where the ball goes and whether the kick was a goal, out, or a save in previous kicks, but not the intended spot towards which the ball was kicked. Hence, as a kicker takes a penalty shot, he has a belief over intended spots of previous kicks. Formally, a belief $(H)$ is a function that maps each information set $H \in H_{i;T_k}$ that Team $k$'s $i$'th kicker's move with positive probability to a probability distribution over histories of actions taken that would lead to the same information set.

## 3.3 Markov Perfection and State-Symmetric Equilibria

Our solution concept is state-symmetric (perfect Bayesian) equilibrium, in which strategies in regular rounds depend only on the state of the game, i.e., on the round number, kicking order, and score difference; strategies in sudden-death rounds depend only on the current kicking order and score difference. The strategies in state-symmetric equilibria are memoryless in that they depend only on the current state.

A perfect Bayesian equilibrium in the game of shootout mechanism is an assessment, i.e., a strategy profile and a belief profile pair $[X = (X_{i;T_{k'}})_{i \in \{1,2,\ldots\},k' \in \{1,2\}}; = ((H))_{H \in H_{i;T_{k'}}; i \in \{1,2,\ldots\}, k' \in \{1,2\}}]$ such that for any $k, \ell \in \{1, 2\}$ s.t. $k \neq \ell$, $i \in \{1, 2, \ldots\}$, and $H \in H_{i;T_k}$;

- spot $X_{i;T_k}(H) \in [0;$

We will determine whether ABAB's equilibria are order independent and inspect other plausible mechanisms by characterizing the class of order-independent mechanisms in regular rounds and providing a large class of order-independent mechanisms in sudden-death rounds.

# 4 Analysis: A Kicker's Optimization Problem

We rst analyze each kicker's optimization problem for a given mechanism and other agents' strategies. The best response determination problem of the i-th kicker of Team k, denoted by (i; $T_k$), boils down to

$$\max_{x \in [0;1]} U(x; W_{G;}; W_{NG;}) \quad P_G(x)W_{G;} + [1 \quad P_G(x)]W_{NG;} \quad + \quad P_G(x)U_G + P_O(x)U_O \qquad (2)$$

where $P_G(x)U_G + P_O(x)U_O$ is Kicker 's expected individual kick payo , and $P_G(x)W_{G;} + [1$ $P_G(x)]W_{NG;}$ is Kicker 's expected continuation team payo given expected continuation values $W_{G;}$ conditional on he scores and $W_{NG;}$ conditional on he does not score. These values, $W_{G;}$ and

is exogenous if, for all rounds $k$, and kicking orders $h^{k-1}$ regarding the beginning of round $k$, $(h^{k-1} : g_{T_1} : g_{T_2}) = (k)$ for some function , i.e., who goes rst in each round is determined

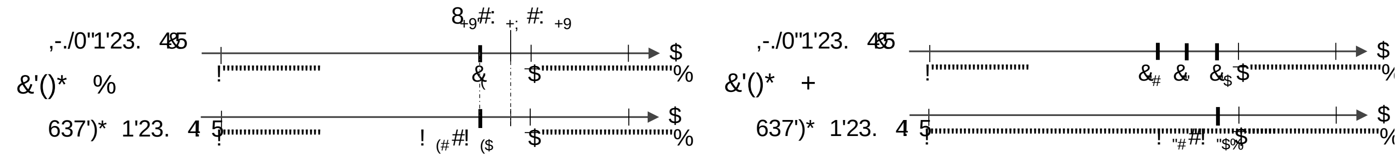

Figure 1: The e ort levels of teams under an order-independent mechanism is state-symmetric equilibria in an easy shootout.

Corollary 1 Let be an order-independent mechanism regardless S... levels, i.e., the state-symmetric equilibrium ... are determined as follows

- In Round 1: (1) for the kicking teams

more aggressive when his team is ahead than behind as we explained previously. (It can also be shown that with a similar argument Kicker 1 of penultimate round has exactly the same marginal contribution as Kicker 2 when this round starts tied under uneven-score symmetric mechanisms.)

In particular, this is exactly why ABAB or ABBA, or any fixed-order mechanism is not order independent: In the last round, as first kicking team is pre-determined and its kicker is more aggressive when it is ahead than behind, the equalities $p^1_A = p^2_A$ and $p^1_B = p^2_B$ do no longer hold (i.e., $\omega_1 = \omega_{1B} = \omega_{1E}$ shown in Figure 1, no longer holds). Hence, even if the penultimate round starts tied, there are possible state-symmetric equilibria in which the kickers of this round will exert different efforts leading to different winning chances for their teams at the beginning of the penultimate round.

The theorem leads to another interesting point: There is only one class of order-independent exogenous mechanisms; the post-Round-1 random-order mechanisms that determine which team will kick first with an unbiased coin toss in each round after the first, while who goes first in Round 1 can be determined freely. We formalize it below, and it follows directly from Theorem 1.

**Proposition 2** *The class of post-Round-1 random-order mechanisms are the only exogenous mechanisms that are order independent.*

Note that one does not need to treat both teams symmetrically all the time to obtain order independence. In fact, when the score is tied, it does not matter which team kicks first. However, when the score is not tied, teams need to be treated symmetrically. This feature opens the door for some interesting practical mechanisms to be order independent. Two subclasses of such mechanisms are the *behind-first* and *ahead-first* mechanisms. In behind first (ahead first), the team that is behind (ahead) in score after a round kicks first in the next round, and otherwise the order of the teams is determined in some other manner. There are also many other uneven score symmetric mechanisms in which lotteries play a significant role. For example, a lottery mechanism that forces the behind team to go first in 75% of the time and also Team 1 always to go first 60% of the time when the score is tied is also order independent.

Next we ask as the sudden-death rounds induce an infinite game, what do order-independent mechanisms look like in sudden-death rounds. It turns out that there order matters when the score is tied unlike in regular rounds.

## 6 Sudden-death Rounds

Sudden-death analysis is substantially different as regular-round analysis assumes that winning chances are equal after they are over and score is still tied, while sudden-death rounds make the game an infinite game and tries to analyze what actually the winning chances are after regular rounds.

Under ABAB or ABBA, one can have uneven scores, such as Team 1 being ahead, in an intermediate regular round. As we showed, however, they cannot satisfy uneven score symmetry of order-independent mechanisms in regular rounds. On the other hand, in the sudden-death rounds, the score is never uneven at the beginning of a round. Suppose sudden death is reached in ABAB and ABBA. Would they at least be order independent in sudden-death rounds? If not, what do sudden-death order-independent mechanisms look like? We start with ABAB to answer these questions.

## 6.1 ABAB in Sudden-death Rounds

We will now characterize the state-symmetric equilibria of ABAB in the sudden-death rounds. As the game is infinite now, we will pedantically take the reader through the kickers' dynamic problem as we did in Section 4 for a single round. Without loss of generality assume that Team 1 wins the coin toss before Round 1 and kicks first throughout.

At state-symmetric equilibria, if they exist, each Team 1 kicker will use exactly the same action when he kicks in the sudden-death rounds, as Team 1 always goes first and the score is tied at the beginning of each sudden-death round. Similarly, by symmetry, each Team 2 kicker will use exactly the same action when his team is behind (which can be by one goal at most), and he will use exactly the same action when the score is even (which can happen if the preceding Team 1 kicker kicks out or his kick is saved).

On the other hand, Team 1 and Team 2 kickers may potentially use different actions at state-symmetric equilibria, as they kick in different orders: in each round Team 1 goes first and Team 2 goes second. Hence, if a state-symmetric equilibrium exists, for a given $k = 1; 2$, the probability of Team $k$ winning is the same at the beginning of each sudden-death round.

At a state-symmetric equilibrium, let us define $V_{T_1}$ to be the value function of Team 1, that is the expected utility it contributes by winning or losing to its all kickers, in the first sudden-death round. Denote by $x$ the kicking strategy for Team 1's kickers. Define $V^B_{T_2}$ as the value function of Team 2 in the first sudden-death round when Team 2 is currently behind by one goal and $V^E_{T_2}$ as the value function of Team 2 in the first sudden-death round when the score is currently even. Team 2's kickers' optimal kicking strategy in each scenario is $y_B$ and $y_E$ respectively.

We can write the following Bellman equation for $V_{T_1}$:

$$V_{T_1} = P_G(x)W_{G;T_1} + [1 \text{ re } 7.5432/F41$$

For Team 2, we have

$$V_{T_2}^B = P_G(y_B) \underbrace{V_{T_2}}_{= W_{G;T_2}^B} + [1 \quad P_G(y_B$$

Actually, for such a restriction to hold, we do not even need the teams to mirror each other's kickers as frequently as in ABBA. In fact, there are uncountably many other mechanisms that are order independent in sudden-death rounds:

**Theorem 4 (Order-independent mechanisms)** *Take any mechanism $\psi$ and any order-independent mechanism $\phi'$. Fix a Sudden-death Round $k$. Construct a mechanism $\phi$ such that for sudden-death rounds before the beginning of Round $k-1$'s end, it uses $\psi$'s order structure, and for any other round, it uses $\phi'$'s order structure. Formally,*

- *for all $\ell$ such that $n < \ell < k$, feasible scores $g_{T_1} : g_{T_2}$, and beginning of Round $\ell$ kicking orders $h^{\ell-1}$, let $\phi(h^{\ell-1}; g_{T_1} : g_{T_2}) = \psi(h^{\ell-1}; g_{T_1} : g_{T_2})$, and*
- *for all $\ell \geq k$ and $\ell \leq n$, feasible scores $g_{T_1} : g_{T_2}$, and beginning of Round $\ell$ kicking orders $h^{\ell-1}$, let $\phi(h^{\ell-1}; g_{T_1} : g_{T_2}) = \phi'(h^{\ell-1}; g_{T_1} : g_{T_2})$.*

*Then $\phi$ is order independent.*

We can use Theorem 4 recursively, to obtain a very large class of order-independent mechanisms. The intuition of this result is as follows: Take the last round before order independence kicks in, say Round $k$. By backward induction, as teams are tied at the beginning of Round $k$ and in Round $k+1$ they have a 50%–50% chance of winning, in all situations the two kickers of Round $k$ exert the same effort regardless of kicking order (as we explained in the intuition behind Theorem 1). Therefore, at the beginning of Round $k$, both teams have an equal chance of winning as well. An example of such a mechanism is a behind-first mechanism such that in the first + 10 rounds Team 1 kicks first whenever the game is tied, and then we alternate the order. Note that in the first 10 sudden-death rounds Team 1 kicks first, and yet, the mechanism is order independent as it is appended by an order-independent mechanism in sudden-death rounds, namely ABBA.

Although state-symmetric equilibria of ABAB in which teams exert different effort are still equilibria of ABBA, these equilibria are no longer state-symmetric under ABBA: If Team 1 kickers always exert a higher effort than Team 2's in ABAB, now their position as first or second kickers will alternate in ABBA. Thus, when the state is “kicking first,” if it is a Team 1 kicker then he will exert higher effort in the same state than Team 2 kicker, violating state symmetry.

## 7 Discussion: Order Independence vs Procedural Fairness

Order independence implies ex-ante fairness, and in our context they are both about the distribution of state-symmetric equilibria. The starting team can be determined by alphabetical order of the names of the teams and yet we can still obtain order independence. Thus, not only an even coin flip to determine which team will start first is not needed, the existence of such coin flip does not guarantee ex-ante fairness of the state-symmetric equilibrium outcomes. That is one other aspect ABAB fails: it is not even ex-ante fair in this sense.

However, there is a certain appeal of procedural fairness that an even coin  ip determines which team will start  rst. This appeal is not only aesthetic: procedural fairness matters, as there are characterize easy shootout mechanisms, such as the ones in football, satisfying order independence and maximization of the expected number of attempts together with the other two properties, namely, simplicity and sudden-death equality of opportunity: The team that is behind in score abnormalities surprisingly well through our approach of one parameter deviation from a model of players with only outcome-oriented preferences.[27]

# A Proofs of Proposition 1, Theorem 1, Corollary 1

Proof of Proposition 1. First observe that $\bar{x}$ solves Equation 4 when $U_O = 0$. As the partial derivative w.r.t. $U_O$ on the (left-hand side of) first-order condition is $P_O^0(x_\cdot) > 0$

By Equation 11, $y_{2E}$ solves the following rst-order condition:

$$P_G^0(y_{2E})\Big[ [P_G(\ )\quad (1\quad P_G(\ ))]\frac{V_W+V_L}{2}+(1\quad P_G(\ ))\,V_W\quad P_G(\ )V_L+U_G\Big]+P_O^0(y_{2E})U_O=0$$

$$=)\quad P_G^0(y_{2E})[\frac{V_W\quad V_L}{2}+U_G]+P_O^0(y_{2E})U_O=0$$

Therefore,

$$y_{2E}= \tag{16}$$

and $V_{T_2;P_2;E}=\frac{V_W+V_L}{2}$:

When Team 2 is currently behind: Let $y_{2B}$ denote the optimal kicking strategy for Team 2's kicker in Round 2 when Team 2 is currently behind. The value function for Team 2 is

$$V_{T_2;P_2;B}=P_G(y_{2B})P_G(\ )V_L+P_G(y_{2B})(1\quad P_G(\ ))\frac{V_W+V_L}{2}+(1\quad P_G(y_{2B}))\,V_L$$

$y_{2B}$ satis es the following rst-order condition:

$$P_G^0(y_{2B})[P_G(\ )V_L+(1\quad P_G(\ ))\frac{V_W+V_L}{2}\quad V_L+U_G]+P_O^0(y_{2B})U_O=0$$

$$=)\quad P_G^0(y_{2B})\ (1\quad P_G(\ ))\frac{V_W\quad V_L}{2}+U_G\ +P_O^0(y_{2B})U_O=0$$

When Team 2 is currently ahead: Let $y_{2A}$ denote the optimal kicking strategy for Team 2's kicker in Round 2 when Team 2 is currently ahead. The value function for Team 2 is

$$V_{T_2;P_2;A}=P_G(y_{2A})V_W+(1\quad P_G(y_{2A}))\ (1\quad P_G(\ ))V_W+P_G(\ )\frac{V_W+V_L}{2}$$

The optimal kicking strategy, $y_{2A}$; satis es the following rst-order condition:

$$P_G^0(y_{2A})[V_W\quad (1\quad P_G(\ ))V_W\quad P_G(\ )\frac{V_W+V_L}{2}+U_G]+P_O^0(y_{2A})U_O=0$$

$$=)\quad P_G^0(y_{2A})[P_G(\ )\frac{V_W\quad V_L}{2}+U_G]+P_O^0(y_{2A})U_O=0 \tag{17}$$

As

$$P_G(\ )>\frac{1}{2}\ =)\quad y_{2A}>y_{2B}: \tag{18}$$

Moreover, since $P_G(\ )<1$, Equations 15 and 17 imply

$$y_{2A}<\ : \tag{19}$$

Case 2: When Team 1 kicks rst in Round 2 Let $x_{2E}$; $x_{2B}$; and $x_{2A}$ denote the optimal kicking strategy for Team 1's kicker in Round 2 when the score is even, when Team 1 is behind, and when Team 1 is ahead respectively. By symmetry, we have the following results:

When the score is currently even: The optimal kicking strategy is

$$x_{2E}=y_{2E}=\ ; \tag{20}$$

where

$$V_{T_2;P_2;B} = P_G(y_{2B})P_G(\ )V_L + P_G(y_{2B})(1 \quad P_G(\ ))\frac{V_W + V_L}{2} + (1 \quad P_G(y_{2B}))V_L$$
$$= \frac{V_W + V_L}{2} \quad 1 \quad P_G(y_{2B})(1 \quad P_G(\ )) \ \frac{V_W \quad V_L}{2}$$
$$V_{T_1;P_2;A} = P_G(x_{2A})V_W + (1 \quad P_G(x_{2A}))\ (1 \quad P_G(\ ))V_W + P_G(\ )\frac{V_W + V_L}{2}$$
$$= \frac{V_W + V_L}{2} + [1 \quad (1 \quad P_G(x_{2A}))P_G(\ )]\frac{V_W \quad V_L}{2}$$

We substitute the equations of $V_{T_2;P_2;B}$ and $V_{T_1;P_2;A}$ into $V_{T_2;P_1;B}$ as follows:

$$V_{T_2;P_1;B} = \frac{V_W + V_L}{2} \quad (1 \quad P_G(y_{1B}))\ \Big[(1 \quad (T_1;1:0))[1 \quad P_G(y_{2B})(1 \quad P_G(\ ))$$
$$+ \ (T_1;1:0)[1 \quad (1 \quad P_G(x_{2A}))P_G(\ )]\Big] \ \frac{V_W \quad V_L}{2}$$

The optimal kicking strategy, $y_{1B}$; satis es the following rst-order condition:

$$P_G^0(y_{1B})[\ _2\frac{V_W \quad V_L}{2} + U_G] + P_O^0(y_{1B})U_O = 0; \tag{23}$$

where

$$_2 = (1 \quad (T_1;1:0))[1 \quad P_G(y_{2B})(1 \quad P_G(\ ))] + \ (T_1;1:0)[1 \quad (1 \quad P_G(x_{2A}))P_G(\ )]$$

Then $y_{1B} = y_{1E}$ i $\ _1 = \ _2$ i

$$(T_1;0:1)[1 \quad P_G(x_{2B})(1 \quad P_G(\ ))] + (1 \quad (T_1;0:1))[1 \quad (1 \quad P_G(y_{2A}))P_G(\ )]$$
$$= (1 \quad (T_1;1:0))[1 \quad P_G(y_{2B})(1 \quad P_G(\ ))] + \ (T_1;1:0)[1 \quad (1 \quad P_G(x_{2A}))P_G(\ )]$$
$$() \quad (1 \quad (T_1;0:1) \quad (T_1;1:0))[1 \quad (1 \quad P_G(y_{2A}))P_G(\ )]$$
$$= (1 \quad (T_1;0:1) \quad (T_1;1:0))[1 \quad P_G(x_{2B})(1 \quad P_G(\ ))]$$
$$() \quad (1 \quad (T_1;0:1) \quad (T_1;1:0))[(1 \quad P_G(y_{2A}))P_G(\ ) \quad P_G(x_{2B})(1 \quad P_G(\ ))] = 0$$

However, $(1 \quad P_G(y_{2A}))P_G(\ ) \quad P_G(x_{2B})(1 \quad P_G(\ )) > 0$ as $\overline{x} > \ > x_{2B}$ and $y_{2A} < \ :$ Accordingly,

$$y_{1B} = y_{1E} \ () \quad (T_1;0:1) + \ (T_1$$

The optimal kicking strategy, $x_1$; satisfies the following first-order condition:

$$P_G^0(x_1)\left[(1 - P_G(y_{1B}))\,_2 + P_G(y_{1E})\,_1\,\frac{V_W - V_L}{2} + U_G\right] + P_O^0(x_1)U_O = 0$$

Therefore

$$x_1 \gtreqless y_{1E} \Leftrightarrow (1 - P_G(y_{1B}))\,_2 \gtreqless (1 - P_G(y_{1E}))\,_1 \tag{25}$$

On the other hand, we have

$$V_{T_1} = \frac{V_W + V_L}{2} \Leftrightarrow P_G(x_1)(1 - P_G(y_{1B}))\,_2 = (1 - P_G(x_1))P_G(y_{1E})\,_1$$

Given that both teams have an equal chance of winning in sudden-death rounds and $V_{T_2;P_2;E} = V_{T_1;P_2;E} = \frac{V_W+V_L}{2}$; is order independent if and only if $V_{T_1} = \frac{V_W+V_L}{2}$: We first make the following claim:

Claim 1. $P_G(x_1)(1 - P_G(y_{1B}))\,_2 = (1 - P_G(x_1))P_G(y_{1E})\,_1$ if and only if $(1 - P_G(y_{1B}))\,_2 = (1 - P_G(y_{1E}))\,_1$:

Proof of Claim 1. ($\Rightarrow$) Suppose to the contrary that $(1 - P_G(y_{1B}))\,_2 \neq (1 - P_G(y_{1E}))\,_1$ but $P_G(x_1)(1 - P_G(y_{1B}))\,_2 = (1 - P_G(x_1))P_G(y_{1E})\,_1$: If $(1 - P_G(y_{1B}))\,_2 > (1 - P_G(y_{1E}))\,_1$; then from the first-order condition of $x_1$ we have $\bar{x} > x_1 > y_{1E}$: Then $P_G(x_1)(1 - P_G(y_{1B}))\,_2 > P_G(y_{1E})(1 - P_G(y_{1B}))\,_2 > P_G(y_{1E})(1 - P_G(y_{1E}))\,_1 > (1 - P_G(x_1))P_G(y_{1E})\,_1$; a contradiction. The other case can be analyzed in a similar fashion.

($\Leftarrow$) If $(1 - P_G(y_{1B}))\,_2 = (1 - P_G(y_{1E}))\,_1$; then from the first-order condition of $x_1$ we have $x_1 = y_{1E}$; which in turn implies

$$P_G(x_1)(1 - P_G(y_{1B}))\,_2 = P_G(y_{1E})(1 - P_G(y_{1B}))\,_2 = P_G(y_{1E})(1 - P_G(y_{1E}))\,_1 = (1 - P_G(x_1))P_G(y_{1E})\,_1$$

Hence the Claim is established.

Accordingly, is order independent if and only if

$$(1 - P_G(y_{1B}))\,_2 = (1 - P_G(y_{1E}))\,_1: \tag{26}$$

This equality holds for an arbitrary pair of (feasible) probabilities, $\{P_G; P_O\}$; if and only if $_1 = \,_2$; which holds if and only if $(T_1; 0:1) + (T_1; 1:0) = 1$; i.e., is uneven score symmetric. ■

Proof of Corollary 1. It can readily be seen from the proof of Theorem 1 that the optimal kicking strategy at that state is solely determined by the state (the score difference and the kicking order in Round 2), and hence is independent of which order-independent mechanism leads to that state. W.l.o.g., suppose Team 1 moves first in Round 1 and Team 2 moves second. Consider Round 1 first. Equation 24 in the proof of Theorem 1 implies that $!_{1E} = !_{1B}$. Equations 25 and 26 imply that $_1 = !_{1E}$. Next consider Round 2. Equation 15 implies that $_{2B} = !_{2E}$. Equations 15 and 16 when Team 1 moves second in Round 2 and Equation 20 when Team 1 moves first in Round 1 imply that $_{2E} = !_{2E}$. For an easy shootout, Equations 18 and 19 when Team 1 moves second in Round 2 and Equations 21 and 22 when Team 1 moves first in Round 2 imply that $_{2E} > \,_{2A} > \,_{2B}$. (where easiness of the shootout is only used in Equation 18, for a difficult shootout, we would have $_{2E} > \,_{2B} > \,_{2A}$): Finally, Equations 15 and 23 and the fact that $_2 < 1$ imply $!_{1B} < !_{2B}$. ■

# B Order Dependence of ABAB in Sudden-death Rounds

Theorem 6 (Order dependence of ABAB in sudden-death rounds) Suppose that in the sudden-death rounds of ABAB a state-symmetric interior equilibrium exists. Then

- Multiple state-symmetric equilibria exist if and only if there are multiple solutions to the equation

$$(\ \ ) \qquad \frac{1 \quad P_G\ y(1 \quad )}{2 \quad P_G\ y(\ ) \quad P_G\ y(1 \quad )} = 0; \tag{27}$$

  where $y(\ ) = f^{\ 1}\ \frac{U_O}{(V_W \quad V_L)\ + U_G}$ for $f(x) = P_G^0(x)/P_O^0(x)$ for all $x \in [0;1]$.

- $(\ \ ) = 0$ has multiple solutions if

$$\frac{U_G}{V_W \quad V_L} < \frac{\ln\ 1 \quad P_G(x)\ ^{0}}{}$$

Thus, ABAB is not order independentas the winning probability of Team 1, $\neq \frac{1}{2}$ in equilibrium, whenever $y_B \neq y_E$.

On the other hand, the su ciency condition in Equation 28

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# Online Appendices for \Designing Practical and Fair Sequential Team Contests"

by Nejat Anbarc , Ching-Jen Sun, & M. Utku Ünver

## C Remaining Proofs of Results

Proof of Theorem 2.

We write the three  rst-order conditions using Equation 11 (or 3) as:

$$P_G^0(x)[P_G(y_B)V_{T_1} + (1 \quad P_G(y_B))V_W \quad P_G(y_E)V_L \quad (1 \quad P_G(y_E))V_{T_1} + U_G] + P_O^0(x)U_O = 0$$

$$P_G^0(y_B)[V_{T_2} \quad V_L + U_G] + P_O^0(y_B)U_O = 0$$

$$P_G^0(y_E)[V_W \quad V_{T_2} + U_G] + P_O^0(y_E)U_O = 0$$

We  rst prove that $x = y_E$ in any state-symmetric equilibrium.

Claim 1. $x = y_E$.

Proof of Claim 1. De ne

$$= \quad P_G(y_B)V_{T_1} + (1 \quad P_G(y_B))V_W \quad P_G(y_E)V_L \quad [1 \quad P_G(y_E)]V_{T_1} \quad V_W + V_{T_2}:$$

From the  rst-order conditions of $x$ and $y_E$, $x \quad y_E$ if and only if $\quad 0$: Recall that the winning probability of Team 1 in equilibrium, , is given in Equation 13. Hence,

$$\begin{aligned} &= \quad P_G(y_B)(V_{T_1} \quad V_W) + P_G(y_E)(V_{T_1} \quad V_L) + V_{T_2} \quad V_{T_1} \\ &= P_G(y_B)(1 \quad )(V_L \quad V_W) + P_G(y_E) \quad (V_W \quad V_L) + (1 \quad 2 \ )(V_W \quad V_L) \\ &= [\quad P_G(y_B)(1 \quad ) + P_G(y_E) \quad + 1 \quad 2 \ ](V_W \quad V_L) \\ &= [1 \quad P_G(y_B) + (\ P_G(y_E) + P_G(y_B) \quad 2) \ ](V_W \quad V_L) \end{aligned}$$

We substitute  from Equation 13 as follows:

$$\begin{aligned} &= [1 \quad P_G(y_B) + (\ P_G(y_E) + P_G(y_B) \quad 2)\frac{P_G(x)(1 \quad P_G(y_B))}{P_G(x)(1 \quad P_G(y_B)) + (1 \quad P_G(x))P_G(y_E)}](V_W \quad V_L) \\ &= (1 \quad P_G(y_B))[1 + \frac{(P_G(y_E) + P_G(y_B) \quad 2)P_G(x)}{P_G(x)(1 \quad P_G(y_B)) + (1 \quad P_G(x))P_G(y_E)}](V_W \quad V_L) \\ &= [\frac{(1 \quad P_G(y_B))(V_W \quad V_L)}{P_G(x)(1 \quad P_G(y_B)) + (1 \quad P_G(x))P_G(y_E)}] \\ &\qquad [P_G(x)(1 \quad P_G(y_B)) + (1 \quad P_G(x))P_G(y_E) + (\ P_G(y_E) + P_G(y_B) \quad 2)P_G(x)] \\ &= \frac{(1 \quad P_G(y_B))(V_W \quad V_L)}{P_G(x)(1 \quad P_G(y_B)) + (1 \quad P_G(x))P_G(y_E)}[P_G(y_E) \quad P_G(x)] \end{aligned}$$

Suppose $x > y_E$, then as both $x; y$

optimal kicking strategy for the rst kicker in each sudden-death round, and $x_B$ ($x_E$) the optimal kicking strategy for the second kicker in each sudden-death round when the score is behind (tied). Let $V_{T_1}$ ($V_{T_2}$) denote Team 1's (Team 2's) value function at the beginning of the rst sudden-death round (Round $n + 1$). Then

$$V_{T_1} = [P_G(x_I)P_G(x_B) + (1$$ [illegible]

Plugging in the expression of  and doing some simpli cations, we have

$$_{IE} = \frac{P_G(x_I)(1 \quad P_G(x_B)) \quad P_G(x_B)(1 \quad P_G(x_E))}{2 \quad (1 \quad P_G(x_I))P_G(x_E) \quad P_G(x_I)(1 \quad P_G(x_B))}(V_W \quad V_L)$$

We can then conclude that $x_I$ T $x_E$ if and only if $x_I$ T $x_B$. Next we compare $x_I$ and $x_B$: De ne

$$_{IB} = [P_G(x_B) \quad (1 \quad P_G(x_E))]V_{T_2} + (1 \quad P_G(x_B))V_W \quad P_G(x_E)V_L \quad (V_{T_1} \quad V_L)$$

as the continuation value under the order-independent mechanism in Round $k$. Suppose $x$ is Team 1's kicker's optimal spot, $y_E$ is Team 2's kicker's optimal spot when they are still tied, and $y_B$ is Team 1's kicker's optimal spot when Team 1 is ahead (by one goal). Recall the first-order conditions through Equation 11 (or 3):

$$P_G'(x)[P_G(y_B)V_{T_1} + (1 - P_G(y_B))V_W - P_G(y_E)V_L - (1 - P_G(y_E))V_{T_1} + U_G] + P_O'(x)U_O = 0$$

$$P_G'(y_B)[V_{T_2} - V_L + U_G] + P_O'(y_B)U_O = 0$$

$$P_G'(y_E)[V_W - V_{T_2} + U_G] + P_O'(y_E)U_O = 0$$

We rewrite Team 2's kicker's first-order conditions plugging in $V_{T_1} = V_{T_2}$:

$$P_G'(y_B)[\frac{V_W - V_L}{2} + U_G] + P_O'(y_B)U_O = 0$$

$$P_G'(y_E)[\frac{V_W - V_L}{2} + U_G] + P_O'(y_E)U_O = 0$$

As $y(\alpha) = f^{-1}\left(\frac{U_O}{(V_W - V_L)\alpha + U_G}\right)$ and $f(x) = P_G'(x)/P_O'(x)$; $y'\left(\frac{1}{2}\right)$ can be computed as:

$$
\begin{aligned}
y'\left(\frac{1}{2}\right) &= \frac{1}{f'(y(\alpha))}\frac{(V_W - V_L)U_O}{((V_W - V_L)\alpha + U_G)^2}\Bigg|_{\alpha=\frac{1}{2}} \\
&= \frac{P_O'(y(\frac{1}{2}))^2}{P_G''(y(\frac{1}{2}))P_O'(y(\frac{1}{2})) - P_O''(y(\frac{1}{2}))P_G'(y(\frac{1}{2}))}\frac{(V_W - V_L)U_O}{\left((V_W - V_L)\frac{1}{2} + U_G\right)^2}.
\end{aligned}
$$

Also we have

$$
y\left(\frac{1}{2}\right) = f^{-1}\left(\frac{U_O}{(V_W - V_L)\frac{1}{2} + U_G}\right) \Longrightarrow (V_W - V_L)\frac{1}{2} + U_G = \frac{P_O'(y(\frac{1}{2}))}{P_G'(y(\frac{1}{2}))}U_O.
$$

Then

$$
\begin{aligned}
y'\left(\frac{1}{2}\right) &= \frac{P_O'(y(\frac{1}{2}))^2}{P_G''(y(\frac{1}{2}))P_O'(y(\frac{1}{2})) - P_O''(y(\frac{1}{2}))P_G'(y(\frac{1}{2}))}\frac{V_W - V_L}{(V_W - V_L)\frac{1}{2} + U_G}\frac{P_G'(y(\frac{1}{2}))}{P_O'(y(\frac{1}{2}))} \\
&= \frac{P_O'(y(\frac{1}{2}))P_G'(y(\frac{1}{2}))}{P_G''(y(\frac{1}{2}))P_O'(y(\frac{1}{2})) - P_O''(y(\frac{1}{2}))P_G'(y(\frac{1}{2}))}\frac{V_W - V_L}{(V_W - V_L)\frac{1}{2} + U_G};
\end{aligned}
$$

and

$$
\phi'(\alpha)|_{\alpha=\frac{1}{2}} = 1 - \frac{P_G'(y(\frac{1}{2}))}{2\left(1 - P_G(y(\frac{1}{2}))\right)}\frac{P_O'(y(\frac{1}{2}))P_G'(y(\frac{1}{2}))}{P_G''(y(\frac{1}{2}))P_O'(y(\frac{1}{2})) - P_O''(y(\frac{1}{2}))P_G'(y(\frac{1}{2}))}\frac{V_W - V_L}{(V_W - V_L)\frac{1}{2} + U_G}.
$$

Therefore $\phi'(\alpha)|_{\alpha=\frac{1}{2}} < 0$ if and only if

$$
\begin{aligned}
& 1 + \frac{2U_G}{V_W - V_L} < \frac{P_G'(y(\frac{1}{2}))}{1 - P_G(y(\frac{1}{2}))}\frac{P_O'(y(\frac{1}{2}))P_G'(y(\frac{1}{2}))}{P_G''(y(\frac{1}{2}))P_O'(y(\frac{1}{2})) - P_O''(y(\frac{1}{2}))P_G'(y(\frac{1}{2}))} \\
\Longleftrightarrow\quad & 1 + \frac{2U_G}{V_W - V_L} < \frac{(\ln(1 - P_G(x)))'}{(\ln f(x))'}\Bigg|_{x=y(\frac{1}{2})}.
\end{aligned}
$$

■

# D Field Evidence

## D.1 Shootout Winning Percentages in Major Tournaments

Figure 2: Empirical Evidence from Table 5.1 in Palacios-Huerta (2014) and Table 1 in Kocher, Lenz, and Sutter (2012): The winning proportions of rst-kicking teams are given on the vertical axis while the numbers of shootouts in the considered championships are given on the horizontal axis. Euro int refers to combined proportion for all European international championships such as European Championship, Champions League, Cup Winners Cup, and UEFA Cup. Observe that as sample size increases (i.e., data points 50 or more) second-mover advantage disappears in major football data tournaments. While there is undisputed rst-mover advantage in Spanish Cup, Euro int and English Cup display somewhat rst-mover advantage, and German Cup displays neither rst- nor second-mover advantage. We also thank Martin Kocher, Marc Lenz, and Matthias Sutter for providing us their data set.

## D.2 ABBA vs ABAB in the Field

Recently ABBA replaced ABAB at the U-17 Women's and Men's World Football Championships. The International Football Association Board (IFAB) decided to implement the ABBA sequence in various trials before eventually using it in Women's World Cup. In addition, the ABBA format for penalty shootouts is adopted in all English Football League (EFL) competitions in 2017-18, and recently in Dutch Cup in 2018-19, while the rest of the world still uses ABAB as of this writing. In dynamic individual contests too, we observe that the ABBA format being utilized. It is used in the U.S. presidential debate sequences. Similarly, FIDE, the governing body of chess, has recently changed the rules for the FIDE World Chess Championship and switched from ABAB to players, i.e., the player has a higher $P_G(x)$ and a lower $P_O(x)$ for every $x \in [0;1]$. We formally define a better player as follows: Let $\{P_G; P_O\}$ represent all players' kicking ability except the better player, and $\{\tilde{P}_G; \tilde{P}_O\}$ represent the better player's kicking ability. We assume (a) $P_G(x) < \tilde{P}_G(x)$ and $P_O(x) > \tilde{P}_O(x)$; and (b) $\frac{P_G^0(x)}{\tilde{P}_G^0(x)} = \frac{P_O^0(x)}{\tilde{P}_O^0(x)}$ for all $x \in [0;1]$. We show that the team with this better player – now named the better team – has a higher winning probability under uneven score symmetric mechanisms.

**Theorem 7** [Uneven team] Suppose a mechanism that is order independent, independent of each round and uneven score symmetric in regard to is used in the shootout. Then a better team has a higher ex-ante chance of winning at the unique subgame-perfect equilibrium of the shootout induced by this mechanism, if the better player is used strategically in the best kicking order possible by the better team.

*Proof of Theorem 7.* We show that by having the better player kick in Round 1, the better team has a higher chance of winning under an uneven score symmetric mechanism. Consider two subcases:

(i) When the better player is in Team 2. Since the better player is placed in Round 1, the second-round maximization problems remain unchanged. Following the proof of Theorem 1, we have $x_{2A} = y_{2A} > x_{2B} = y_{2B}$; and the last kicker's optimal kicking strategy is : Next we study the second team's optimal kicking strategy in Round 1. When Team 1 does not score in Round 1, the value function for Team

The optimal kicking strategy, $y_{1E}$; satis es the following  rst-order condition:

$$\tilde{P}'_G(y_{1E})[\ _1\frac{V_W \quad V_L}{2} + U_G] + \tilde{P}'_O(y_{1E})U_O = 0; \text{ where}$$

$$_1 = \quad (T_1; 0:1)[1 \quad P_G(x_{2B})(1 \quad P_G(\ ))] + (1 \qquad (T_1; 0:1))[1 \quad (1 \quad P_G(y_{2A}))P_G(\ )]:$$

When Team 1 scores in Round 1, the value function for Team 2 is

$$V_{T_2;P_1;B} = \tilde{P}_G(y_{1B})\frac{V_W + V_L}{2} + (1 \quad \tilde{P}_G(y_{1B}))\ (1 \quad (T_1; 1:0))V_{T_2;P_2;B} + \quad (T_1; 1:0)(V_W + V_L \quad V_{T_1;P_2;A})\ ;$$

where

$$V_{T_2;P_2;B} = P_G(y_{2B})P_G(\ )V_L + P_G(y_{2B})(1 \quad P_G(\ ))\frac{V_W + V_L}{2} + (1 \quad P_G(y_{2B}))V_L$$

$$= \frac{V_W + V_L}{2} \quad [1 \quad P_G(y_{2B})(1 \quad P_G(\ ))]\frac{V_W \quad V_L}{2}$$

$$V_{T_1;P_2;A} = P_G(x_{2A})V_W + (1 \quad P_G(x_{2A}))[(1 \quad P_G(\ ))V_W + P_G(\ )\frac{V_W + V_L}{2}]$$

$$= \frac{V_W + V_L}{2} + [1 \quad (1 \quad P_G(x_{2A}))P_G(\ )]\frac{V_W \quad V_L}{2}$$

We substitute the equations of $V_{T_2;P_2;B}$ and $V_{T_1;P_2;A}$ into $V_{T_2;P_1;B}$ as follows:

$$V_{T_2;P_1;B} = \frac{V_W + V_L}{2} \quad (1 \quad \tilde{P}_G(y_{1B}))\Big[(1 \quad (T_1; 1:0))[1 \quad P_G(y_{2B})(1 \quad P_G(\ ))]$$
$$+ \quad (T_1; 1:0)[1 \quad (1 \quad P_G(x_{2A}))P_G(\ )]\Big]\frac{V_W \quad V_L}{2}$$

The optimal kicking strategy, $y_{1B}$; satis es the following  rst-order condition:

$$\tilde{P}'_G(y_{1B})\left[\Big[(1 \quad (T_1; 1:0))[1 \quad P_G(y_{2B})(1 \quad P_G(\ ))]\right.$$
$$\left. + \quad (T_1; 1:0)[1 \quad (1 \quad P_G(x_{2A}))P_G(\ )\Big]\frac{V_W \quad V_L}{2} + U_G\right] + \tilde{P}'_O(y_{1B})U_O = 0$$

Given that $y_{2B} = x_{2B}$ and $x_{2A} = y_{2A}$; the  rst-order condition can be rewritten as

$$P'_G(y_{1B})[\ _2\frac{V_W \quad V_L}{2} + U_G] + P'_O(y_{1B})U_O = 0; \text{ where}$$

$$_2 = (1 \qquad (T_1; 1:0))[1 \quad P_G(x_{2B})(1 \quad P_G(\ ))] + \quad (T_1; 1:0)[1 \quad (1 \quad P_G(y_{2A}))P_G(\ )]:$$

Under an order-independent mechanism, $(T_1; 0:1) + \quad (T_1; 1:0) = 1$; and we have $_1 = \quad _2$: Accordingly, $y_{1E} = y_{1B}$: Finally, we solve for Team 1's optimal kicking strategy in Round 1. The value function for Team 1 is

$$V_{T_1} = P_G(x_1)[V_W + V_L \quad V_{T_2;P_1;B}] + (1 \quad P_G(x_1))[V_W + V_L \quad V_{T_2;P_1;E}]$$
$$= V_W + V_L \quad P_G(x_1)V_{T_2;P_1;B} \quad (1 \quad P_G(x_1))V_{T_2;P_1;E}$$
$$= \frac{V_W + V_L}{2} + [P_G(x_1)(1 \quad \tilde{P}_G(y_{1B}))\ _2 \quad (1 \quad P_G(x_1))\tilde{P}_G(y_{1E})\ _1]\frac{V_W \quad V_L}{2}$$

The optimal kicking strategy, $x_1$; satisfies the following first-order condition:

$$P_G^0(x_1)\left[(1 - \tilde{P}_G(y_{1B}))\,_2 + \tilde{P}_G(y_{1E})\,_1\,\frac{V_W - V_L}{2} + U_G\right]$$

condition, the three optimal kicking strategies in Round 1 are the same $x_1 = y_{1E} = y_{1B}$ (see Corollary 1) and they are determined by the following first-order condition:

$$P_G^0(x_1)[\ _1 \frac{V_W \quad V_L}{2} + U_G] + P_O^0(x_1)U_O = 0; \text{ where}$$

$$_1 = \ (T_1; 1:0)[1 \quad (1 \quad P_G(y_{2A}))P_G(\ )] + (1 \quad (T_1; 1:0))[1 \quad P_G(x_{2B})(1 \quad P_G(\ ))]:$$

Hence the higher the value of $_1$; the higher $x_1$. As $x_{2B} <$ , which is Round 2 second kicking team's intended spot, and $y_{2A} <$ ; we obtain $1 \quad P_G(x_{2B})(1 \quad P_G(\ )) > 1 \quad (1 \quad P_G(y_{2A}))P_G(\ )$: Therefore maximum $x_1$ is achieved in an order-independent mechanism when $(T_1; 1:0) = 0$; i.e., when is a behind-first mechanism. On the other hand, minimum $x_1$ is achieved when $(T_1; 1:0) = 1$; i.e., when is an ahead-first mechanism. ■

The intuition behind this result can be summarized as follows for behind first (ahead first is symmetric). First we summarize the incentives facing Round 2 kickers. In Round 2, kicking first is not good at all for higher goal efforts: the first-kicking team's player (if his team is either behind or ahead) will always exert less effort than he would in the case when he kicks second in Round 2. This is true because his marginal contribution will be less in the first case, as the other team's kicker – who will go second – can always miss or offset the first kicker's failure. So he has higher incentives to shirk when he kicks first. Now, we turn our attention to Round 1 kickers' marginal contributions under both mechanisms. First, observe that both teams' kickers under any uneven score symmetric mechanism exert the same effort in Round 1, by Corollary 1. Therefore, understanding the first-kicking team player's incentives is sufficient to draw the difference between the two mechanisms regardless of the kicking order or score during Round 1. A Round 1 kicker, if he does not exert high effort under behind first, may cause his team to fall behind with higher probability. This causes his teammate to shirk more, when he goes first, and the other team's second player to exert higher effort, when he goes second in Round 2. On the other hand, under ahead first, the Round 1 kicker's incentives are exactly the opposite! If he does not exert high effort in Round 1, his team may fall behind with higher probability, but his teammate will exert relatively higher effort under ahead first by going second in Round 2 (with respect to behind first) and the other team's second kicker will exert less effort in Round 2 (with respect to behind first). Hence, Round 1 kicker's possible failure can still be salvaged with higher probability under ahead first. So he shirks under ahead first vis-a-vis behind first. Therefore, behind first dominates any random (i.e., convex combination of ahead first and behind first) and ahead-first mechanisms among all uneven score symmetric mechanisms. On the other hand, observe that ahead first and behind first cannot be compared with each other in Round 2 whenever the score is not tied: in ahead first when Team 1 is ahead, Team 1 kicks first while in behind first, it kicks second under the same scenario. So there are no two comparable information sets that are reached with positive probability under both mechanisms in Round 2. When the score is tied however, all uneven score symmetric mechanisms lead to the same goal efforts and are equivalent in Round 2. Thus, round by round we are not able to establish an effort ranking among different order-independent shootout mechanisms. Nevertheless, we can still obtain

does not take a kick if the ahead team moves first and scores or the behind team moves first and misses. Given that in an easy shootout the probability of scoring is higher than the probability of missing, this probability is minimized under behind first. Hence, overall, behind first maximizes the expected number of attempts among all order-independent mechanisms. The intuition is reversed for difficult shootouts. Although behind-first mechanisms have nice features when the score is uneven, as mentioned before they are silent on how to define the kicking order when the score is tied. Order independence in regular rounds, by our characterization in Theorem 1, is also mute on this issue, but reversing the kicking order is a sure way of establishing order independence in sudden-death rounds (Theorem 3). ABBA, which is not order independent in regular rounds since it does not satisfy uneven score symmetry, does possess a nice property: When the score is tied in most crucial rounds, i.e., in sudden-death rounds, it gives both teams an equality of opportunity of kicking first. Clearly, such an equality-of-opportunity property is nowhere more important than mechanism in sudden-death rounds: Team 1 kicks first in Round $r$ as long as the score is tied or Team 1 is behind in Round $r - 1$; once Team 2 falls behind after some Round $r' > r$, Team 2 kicks first until Team 1 falls behind in score after some Round $r'' > r'$, after which Team 1 kicks first. One can improve on such a patchy mechanism by requiring that such an eclecticism should be eliminated. We will introduce two properties such that the latter uses the former in its definition to formalize this intuition of simplicity. Before introducing the first property, we formally introduce how an order pattern can be recognized in a mechanism:

A *finite machine representation* of a mechanism is a triple $(Q; A; t)$ such that

- $Q$ is a finite set of (machine) *states* such that state $q = (T_k)_w \in Q$ denotes that Team $k$ taking the first penalty shot in the round associated with this state and $w$ is just an index number. Thus, $Q$ can be partitioned into two as $Q_{T_1} = \{(T_1)_1; \ldots; (T_1)_{w_1}\}$ and $Q_{T_2} = \{(T_2; \ldots; (T_2)_{w_2}\}$ for some $w_1$ and $w_2$ as the sets of states in which Team 1 and Team 2 kick first, respectively.

- $A = \{(g_1 : g_2)\}$ is the set of *possible scores*.

- $t : Q \cup \{\emptyset\} \times A \times Q \to [0; 1]$ is a *state transition probability function* such that $\sum_{q' \in Q} t(q; (g_1 : g_2); q') = 1$ for all $q \in Q \cup \{\emptyset\}$ and $(g_1 : g_2) \in A$. Here, $t(q; (g_1 : g_2); q')$ is the probability of moving from state $q$ to state $q'$ when after round associated with $q$ is played and the score is $g_1 : g_2$ just before $q'$ and after $q$.

We refer to null state $\emptyset$, as the *start of the shootout* di erences are the same. For example, the alternating-order behind- rst mechanism has this type of a representation as shown in Figure 3. We state the following proposition whose proof is given

Figure 3: The state transition representation for the alternating-order behind- rst mechanism. Transitions from the start of the shootout are omitted for simplicity. In general one of the two states in the gure will be chosen randomly with an unbiased lottery. Alternating-order ahead rst's representation is symmetrically de ned.

in the gure for behind rst:

Proposition 5 The alternating-order behind- rst (ahead- rst) mechanism is stationary.

Machine representations can be used to measure the complexity of an algorithm.[35] However, very complicated mechanisms can also be stationary.[36] On the other hand, if we would like to have a chance of both teams kicking rst in at least one round, we need at least two states, one Team-1-kicking- rst state and one Team-2-kicking- rst state. Thus, jQj during a game necessitates replay of the game. Shootout mechanisms that satisfy the simplicity property will make the process easier to administer for the referees and will make the process less prone to rule violations. We see simplicity as a vital requirement of a real-life shootout mechanism. The current mechanism satisfies simplicity but none of the other properties we have introduced in this paper. We formalize the simplicity of the alternating-order behind first (ahead first) with the following proposition. We gave its proof earlier through Figure 3:

**Proposition 6** *The alternating-order behind-first (ahead-first) mechanism is simple.*

We state the main result of this appendix as follows (which was stated as Theorem 5 in Discussion section of the main text).

**Theorem 8** *In an easy shot, alternating-order behind-first is the unique order-independent mechanism that maximizes the expected number of attempts and satisfies simplicity and sudden-death equality of opportunity. On the other hand, in a difficult shot, alternating-order ahead-first is the unique order-independent mechanism that maximizes the expected number of attempts and satisfies simplicity and sudden-death equality of opportunity.*

*Proof of Theorem 8.* Observe that the mechanisms that satisfy the properties should be behind-first, since behind-first mechanisms are the only ones that satisfy order independence and maximizing expected number of attempts (by Proposition 4). The mechanisms that satisfy the sudden-death equality of opportunity (SDEO from now on) have to have each team kicking first in every two sudden-death rounds exactly once. Hence, the only kicking order that is simple and SDEO in the sudden-death rounds is alternating-order. Stationarity (as implied by simplicity) implies that the order of kicking switches when the score stays even between two rounds — i.e., if the state was reached after a tie in score, the order switches after this state if the tied score continues. But this does not imply how the kicking order changes if we transition to a tied score from an uneven score. Simplicity implies that we have two states as $Q = \{(T_1)_1; (T_2)_1\}$. Thus, we need to use the same states of sudden-death rounds also in the regular rounds. Hence, as kicking order switches when the score is tied, i.e. we transition from $(T_1)_1$ to $(T_2)_1$ or the other way around in the sudden-death that satis es all properties but is not simple is a Prouhet-Thue-Morse behind- rst (ahead- rst) mechanism for easy (di cult) shootouts.

Finally, an interesting and relevant question is whether the behind- rst feature has been used in real life. Perhaps it is nowhere more blatant and e ectively at work than in the rules of

where

$$ {}_{2;2;(1;1)} = \quad {}_3(1;2) \quad {}_{3;1;(1;2)} + (1 \qquad {}_3(1;2)) \quad {}_{3;1;(2;1)} $$

The optimal kicking strategy, $x_{2;2;(1;1)}$; satis es the following  rst-order condition:

$$ P_G^0(x_{2;2;(1;1)})[ \quad {}_{2;2;(1;1)} \frac{V_W \quad V_L}{2} + U_G] + P_O^0(x_{2;2;(1;1)})U_O = 0 $$

When $s = (2\,;1)$; the value function for the kicker is

$$ \begin{aligned} V_{2;2;(2;1)} &= P_G(x_{2;2;(2;1)})\frac{V_W + V_L}{2} + (1 \quad P_G(x_{2;2;(2;1)}))[ \quad {}_3(2;1)(V_W + V_L \quad V_{3;1;(2;1)}) + (1 \qquad {}_3(2;1))V_{3;1;(1;2)}] \\ &= \frac{V_W + V_L}{2} \quad (1 \quad P_G(x_{2;2;(2;1)})) \quad {}_{2;2;(2;1)} \frac{V_W \quad V_L}{2}; \end{aligned} $$

where

$$ {}_{2;2;(1;0)} = \quad {}_3(2;1) \quad {}_{3;1;(2;1)} + (1 \qquad {}_3(2;1)) \quad {}_{3;1;(1;2)}: $$

The optimal kicking strategy, $x_{2;2;(2;1)}$; satis es the following  rst-order condition:

$$ P_G^0(x_{2;2;(2;1)})[ \quad {}_{2;2;(1;0)} \frac{V_W \quad V_L}{2} + U_G] + P_O^0(x_{2;2;(2;1)})U_O = 0 $$

When $s = (0\,;1)$; the value function for the kicker is

$$ \begin{aligned} V_{2;2;(0;1)} &= P_G(x_{2;2;(0;1)})V_W + (1 \quad P_G(x_{2;2;(0;1)}))[ \quad {}_3(0;1)(V_W + V_L \quad V_{3;1;(0;1)}) + (1 \qquad {}_3(0;1))V_{3;1;(1;0)}] \\ &= \frac{V_W + V_L}{2} + \quad {}_{2;2;(0;1)} \frac{V_W \quad V_L}{2}; \end{aligned} $$

where

$$ {}_{2;2;(0;1)} = P_G(x_{2;2;(0;1)}) + (1 \quad P_G(x_{2;2;(0;1)}))[ \quad {}_3(0;1) \quad {}_{3;1;(0;1)} + (1 \qquad {}_3(0;1)) \quad {}_{3;1;(1;0)}]: $$

The optimal kicking strategy, $x_{2;2;(0;1)}$; satis es the following  rst-order condition:

$$ P_G^0(x_{2;2;(0;1)}) f [1 \quad [ \quad {}_3(0;1) \quad {}_{3;1;(0;1)} + (1 \qquad {}_3(0;1)) \quad {}_{3;1;(1;0)}]] \frac{V_W \quad V_L}{2} + U_G g + P_O^0(x_{2;2;(0;1)})U_O = 0 $$

When $s = (2\,;0)$; the value function for the kicker is

$$ \begin{aligned} V_{2;2;(2;0)} &= P_G(x_{2;2;(2;0)})[ \quad {}_3(2;1)(V_W + V_L \quad V_{3;1;(2;1)}) + (1 \qquad {}_3(2;1))V_{3;1;(1;2)}] + (1 \quad P_G(x_{2;2;(2;0)}))V_L \\ &= \frac{V_W + V_L}{2} \qquad {}_{2;2;(2;0)} \frac{V_W \quad V_L}{2}; \end{aligned} $$

where

$$ {}_{2;2;(2;0)} = P_G(x_{2;2;(2;0)})[ \quad {}_3(2;1) \quad {}_{3;1;(2;1)} + (1 \qquad {}_3(2;1)) \quad {}_{3;1;(1;2)}] + 1 \quad P_G(x_{2;2;(2;0)}): $$

The optimal kicking strategy, $x_{2;2;(2;0)}$; satis es the following  rst-order condition:

$$ P_G^0(x_{2;2;(2;0)}) f [1 \quad [ \quad {}_3(2;1) \quad {}_{3;1;(2;1)} + (1 \qquad {}_3(2;1)) \quad {}_{3;1;(1;2)}]] \frac{V_W \quad V_L}{2} + U_G g + P_O^0(x_{2;2;(2;0)})U_O = 0 $$

Round 2, First Kick. When $s = (0;0)$ or $s = (1;1)$; the value function for the team is $\frac{V_W + V_L}{2}$: When $s = (0;1)$, the value function for the kicker is

$$V_{2;1;(0;1)} = P_G(x_{2;1;(0;1)})(V_W + V_L \quad V_{2;2;(1;1)}) + (1 \quad P_G(x_{2;1;(0;1)}))(V_W + V_L \quad V_{2;2;(0;1)})$$
$$= \frac{V_W + V_L}{2} \quad {}_{2;1;(0;1)} \frac{V_W \quad V_L}{2};$$

where

$$ {}_{2;1;(0;1)} = P_G(x_{2;1;(0;1)})P_G(x_{2;2;(1;1)}) \quad {}_{2;2;(1;1)} + (1 \quad P_G(x_{2;1;(0;1)})) \quad {}_{2;2;(0;1)}:$$

where

$$_{1;2;(1;0)} = \quad _2(1;0) \quad _{2;1;(1;0)} + (1 \quad _2(1;0)) \quad _{2;1;(0;1)}:$$

The optimal kicking strategy, $x_{1;2;(1;0)}$; satis es the following rst-order condition:

$$P_G^0(x_{1;2;(1;0)})[\quad _{1;2;(1;0)} \frac{V_W \quad V_L}{2} + U_G] + P_O^0(x_{1;2;(1;0)})U_O = 0:$$

Round 1, First Kick. The value function for the kicker is

$$\begin{aligned} V_{1;1;(0;0)} &= P_G(x_{1;1;(0;0)})[V_W + V_L \quad V_{1;2;(1;0)}] + (1 \quad P_G(x_{1;1;(0;0)}))[V_W + V_L \quad V_{1;2;(0;0)}] \\ &= \frac{V_W + V_L}{2} + [P_G(x_{1;1;(0;0)})(1 \quad P_G(x_{1;2;(1;0)})) \quad _{1;2;(1;0)} \\ &\quad (1 \quad P_G(x_{1;1;(0;0)}))P_G(x_{1;2;(0;0)}) \quad _{1;2;(0;0)}]\frac{V_W \quad V_L}{2} \end{aligned}$$

The optimal kicking strategy, $x_{1;1;(0;0)}$; satis es the following rst-order condition:

$$P_G^0(x_{1;1;(0;0)}) \left[ (1 \quad P_G(x_{1;2;(1;0)})) \quad _{1;2;(1;0)} + P_G(x_{1;2;(0;0)}) \quad _{1;2;(0;0)} \frac{V_W \quad V_L}{2} + U_G \right] + P_O^0(x_{1;1;(0;0)})U_O = 0$$

Therefore

$$x_{1;1;(0;0)} \gtreqless x_{1;2;(0;0)} \quad () \quad (1 \quad P_G(x_{1;2;(1;0)})) \quad _{1;2;(1;0)} \gtreqless P_G(x_{1;2;(0;0)}) \quad _{1;2;(0;0)}$$

On the other hand, we have

$$\begin{aligned} V_{1;1;(0;0)} = \frac{V_W + V_L}{2} \quad () \quad & P_G(x_{1;1;(0;0)})(1 \quad P_G(x_{1;2;(1;0)})) \quad _{1;2;(1;0)} = (1 \quad P_G(x_{1;1;(0;0)}))P_G(x_{1;2;(0;0)}) \quad _{1;2;(0;0)} \\ () \quad & (1 \quad P_G(x_{1;2;(1;0)})) \quad _{1;2;(1;0)} = (1 \quad P_G(x_{1;1;(0;0)})) \quad _{1;2;(0;0)} \end{aligned}$$

The condition holds if $_2(1;0) + \quad _2(0;1) = 1$:

# H First-Mover Advantage: A Re nement

Here, we address the question as to which state-symmetric equilibrium is more likely to be observedr8[(W)]T equilibrium, this would be the most bene cial for Team 1. In this case, we can use such a signaling through beliefs in the state-symmetric equilibrium to obtain a re nement. For example, if $_x$ , the probability density function of the ball reaching a particular spot on the goal line when it is aimed at x has the support set [x $_x$