Measuring the Return to Online Advertising: Estimation and Inference of Endogenous Treatment E ects

1 Introduction

[illegible]

[illegible] *caused* [illegible]

Y . [illegible]

[illegible] V [illegible]

[illegible] [illegible] U . [illegible]

[illegible]

$$Y = D + f(\ ) + U$$
$$D = 1\{g(\ ) \quad V \quad 0\};$$

[illegible] f ( ) [illegible] g ( ) [illegible] (U; V) [illegible]

[illegible] E[U j ;]=0. [illegible] .

[illegible]

[illegible] U [illegible] V . [illegible]

[illegible] (2011) [illegible] (2014) D [illegible]

[illegible]

[illegible] eg [illegible] (2019), [illegible] (2014) [illegible]

[illegible] (2016) [illegible]

[illegible]

[illegible] . [illegible]

[illegible] [illegible] X = f ( ) [illegible] Z = g( ). [illegible]

[illegible] . [illegible]

$$(Y \quad X)D = \quad D + UD;$$

[illegible]

$$E[(Y \quad X)D \mid X; Z] = \quad F_V(Z) + E[U1\{V \quad Z\} \mid X; Z]:$$

[illegible]

$$E[Y \mid \ ; \ ] = \ P \ (Z) + X + E[U \mid \ ; \ ] = \ P \ (Z) + f \ ( ):$$

[illegible]

$$E[Y \mid {}^{0}; \ ] \quad E[Y \mid \ ; \ ] = f \ ({}^{0}) \quad f \ ( );$$

[illegible]

$$E[Y \mid \ ; \ ] = E[Y \mid \ ; Z] = \ P \ (Z) + f \ ( ):$$

[illegible]

$$f \ ( ) = \lim_{Z \to \ 1} E[Y \mid \ ; Z]:$$

[illegible]

$$= \frac{\lim_{Z \to +1} E[Y \mid X; Z] \quad \lim_{Z \to \ 1} E[Y \mid X; Z]}{\lim_{Z \to +1} P \ (Z) \quad \lim_{Z \to \ 1} P \ (Z)}: \qquad (\ .1)$$

3 Fisher information for causal e ect

[illegible] *quality* [illegible] $f \ ()$ [illegible] $g \ ()$ [illegible] ' [illegible] 19) [illegible] 19 3)). [illegible] $U; V)$ [illegible] $X; Z)$ [illegible]

Assumption 1 *(i) The index variables* X *and* Z *have a joint distribution where* Z *has a full support on* R *with the joint support not contained in any proper one-dimensional subspace. The parameter of interest is in the interior of a convex compact set* A*;*

(ii) $(U; V)$ *have an absolutely continuous density conditional on* X *and* Z *with full support on* R^2*;*

(iii) The conditional density of

[illegible]

[illegible]

[illegible]

[illegible]

$$b = \frac{1}{n(n-1)} \sum_{i \neq j} (\hat{Y}_i - \hat{Y}_j) D_i (1 - D_j) 1\{ \hat{P}_i - \hat{P}_j | < g; \qquad (4.)$$

[illegible] D_i [illegible] i, $\hat{Y}_i$ [illegible] $\hat{P}_i$ [illegible]

5 Simulation Study

[illegible]

$$d_i = I[z_i' \ _0 \quad _i > 0]$$
$$y_i = \ _0 d_i + x_i' \ _0 + \ _i$$

[illegible] $d_i; y_i$ [illegible] $z_i; x_i$, [illegible]

[illegible]

$_0$, [illegible] $_0$; $_0$ [illegible]

[illegible] $_0$.

[illegible] $x_i; z_i$ [illegible]

[illegible] $E[d_i/z_i]$ [illegible]

[illegible] $P(z_i)$ [illegible]

[illegible] $y_i \quad d_i \; _0 + f(x_i) + \; _i$. [illegible]

[illegible] $x_i; z_i$ [illegible]

[illegible] $_i$;

[illegible]

[illegible]

[illegible] ji ii jj j) to [illegible] $ii; jj$ [illegible] 100, [illegible] x_i to z_i [illegible]. [illegible] 4000. [illegible]

[illegible] " . [illegible] 3 6 [illegible] " [illegible] " [illegible] 4 . [illegible] 3 6 [illegible] 3 6 [illegible] 10 [illegible] 4 [illegible] 1 . [illegible] 1 . [illegible] 3 6 [illegible] 3 [illegible] 10000 [illegible] 1, , 4 , [illegible]

	n = 50	n = 100	n = 200	n = 400	n = 800
= 0					
	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0
= 0 : 2					
	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0
= 0 : 5					
	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0
= 0 : 7					
	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0

	n = 50	n = 100	n = 200	n = 400	n = 800
= 0					
	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0
= 0 : 2					
	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0
= 0 : 5					
	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0
= 0 : 7					
	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0

	n = 50	n = 100	n = 200	n = 400	n = 800
= 0					
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
= 0 :2					
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
= 0 :5					
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
= 0 :7					
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0

	n = 50	n = 100	n = 200	n = 400	n = 800
= 0					
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
= 0 :2					
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
= 0 :5					
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
= 0 :7					
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0

	n = 0	n = 0	n = 0	n = 0
= 0 :5				
	0	0	0	0
	0	0	0	0
	0	0	0	0
	0	0	0	0

6 Empirical Context and Results

[illegible]

[illegible] (2010)). [illegible]

[illegible] (2011). [illegible]

[illegible]

[illegible] (2014)). [illegible]

[illegible] 2014 [illegible]

[illegible] 1, [illegible] 3 . [illegible]

[illegible] 2014 [illegible]

[illegible]

[illegible]

[illegible] 3 [illegible] 3 [illegible]

bidders that met the reserve price to ll the 3 slots, then fewer than 3 ads were shown. We carefully control for this issue by selecting only queries that matched into bidding data where an ad would have been shown in the absence of the experiment.

C Summary statics

C.1 Summary statistics of raw variables

Below we report raw values of the variables that were used to form explanatory variables for the models of the propensity score and the model of the conditional expectation of the outcome variable. Our entire sample contains 9,318,608 observations. To generate variables for LASSO we used interactions and polynomials of variables up to degree 3. That creates an overall of 8,436 variables that were used in the LASSO step. The \Request time" variable is collected but not reported to avoid revealing the exact timing of the experiment.

[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	0	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	0	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	0	0	0	0	[illegible]
[illegible]	[illegible]	[illegible]	0	0	0	0	[illegible]
[illegible]	[illegible]	[illegible]	0	0	0	0	[illegible]
[illegible]	[illegible]	[illegible]	0	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	0	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	0	0	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	0	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	0	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	0	0	0	0	[illegible]
[illegible]	[illegible]	[illegible]	0	[illegible]	0	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	0	0	0	0

C.2 Summary of the LASSO step

Our online appendix lists our generated variables and shows which variables were selected at the LASSO step for the propensity score equation and for the outcome equation. The demonstrated results are provided for our entire dataset that contains an overall of 350 top brands (by search volume) and a subset of the top 50 brands.

For the entire dataset LASSO selected 2,166 variables for the propensity score and 2,114 variables for the outcome equation (out of 8,436). For the subset of top 50 brands LASSO selected 1,907 variables for the propensity score equation and 1,954 for the outcome equation.