Leverage e ects and stochastic volatility in spot oil returns: A Bayesian approach with VaR and CVaR applications[y]

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Abstract

1. Introduction

Crude oil markets have been quite volatile and risky in the past few decades due to the large fluctuations of oil prices. This has become a principal concern for oil suppliers, oil consumers, relevant firms and governments. In addition, as a primary source of energy in the power industry, industrial production and transportation, volatile oil prices may lead to cost uncertainties for other markets, thus extensively affecting the development of the economy. A large number of studies have shown that oil price fluctuations could have considerable impact on economic activities. Papapetrou (2001) argues that the variability of oil prices plays a critical role in affecting real economic activity and employment. Lardic and Mignon (2008) explore the long-term relationship between oil prices and GDP, and find evidence that aggregate economic activity seems to slow down particularly when oil prices increase. This asymmetry is found in both the U.S. and European countries. Consequently, quantifying and managing the risks inherent to the volatility of oil prices has become critical for both researchers and energy market participants.

The Value at Risk (VaR) measure, which was first proposed by J.P. Morgan in the RiskMetrics model in 1994, has been developed as one of the most popular approaches in financial markets to manage market risk. VaR defines the maximum amount that an investor can face for a given tolerance level over a certain time horizon. Although VaR is recommended by Basel II and III and has been widely adopted by financial institutions, it has been challenged by the Bank of International Settlements (BIS) Committee, who pointed out that VaR cannot measure market risk as it fails to consider the extreme tail events of a return distribution (see, Chen et al., 2012). In addition, Artzner et al. (1999) argue that an MA component) are able to replicate the main features of the data more efficiently than GARCH models. At the same time, they find a significant negative leverage effect in crude oil spot markets. Kristoufek (2014) focuses on the leverage effect in commodity futures markets and provides an extensive literature review in this area. Fan et al. (2008)

2. Stochastic volatility models

We use a general SV model to capture the volatility features for oil markets which has been studied recently by Takahashi et al. (2009), Chai et al. (2011) and Chan et al. (2016a):

$$y_t = \quad + \quad {}_t z_t \tag{1}$$

$$ln \ {}^2_t = h_t = \quad + \quad (ln \ {}^2_{t\ 1} \quad) + \ {}_t \qquad {}_t \quad N(0; \ {}^2) \tag{2}$$

where y_t denotes stock returns at time t with $t = 1; 2; :::; T$, denotes the conditional mean, $_t$ is the stochastic volatility, $ln \ {}^2_t$ follows a stationary AR(1) process with persistence parameter having $j \ j < 1$, z_t and $_t$ represent a series of independent identical ($i:i:d:$) random errors in the return and volatility equation, respectively.[2]

For this general equation, we consider various possible speci cations of the shocks z_t a ecting stock returns.

(1) Standard Student t errors

$$z_t \quad t$$

where is the degrees of freedom of t-distribution.

(2) Standard Normal errors

$$z_t \quad N(0; 1)$$

(3) Standard Asymmetric Laplace errors

$$z_t \quad ALD(0; \ ; 1)$$

where = 1 and is the coe cient driving the skewness of the distribution, is related to and as follows:

$$= \frac{}{\sqrt{2}} \quad \frac{1}{}$$

as a special case = 1 for ' 0 and $= e^{\frac{h_t}{2}} > 0$ (*Symmetric Laplace Distribution*).[3]

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[2]A number of original empirical works via extended SV models can be found from Breidt et al. (1998), So et al. (1998), Yu and Yang (2002), Koopman and Uspensky (2002), Cappuccio et al. (2004), Chan (2013), Chan and Hsiao (2013), Chan and Grant (2016c), Chan (2017).

[3]See appendix for the density of ALD.

(4) Standard Student t errors with leverage effect

$$
\begin{aligned}
y_t &= \mu + \sigma_t z_t \\
z_t &\sim t_\nu \\
\ln \sigma_t^2 = h_t &= \alpha + \phi\left(\ln \sigma_{t-1}^2 - \alpha\right) + \eta_t \\
\eta_t &= \rho z_t + \sqrt{1-\rho^2}\,\varepsilon_t \\
\varepsilon_t &\sim N\left(0; \sigma_\eta^2\right)
\end{aligned}
$$

where the coefficient ρ drives the so called *leverage effect*. It models the correlation between the shocks affecting returns and the shocks affecting volatility. For example, a negative ρ would mean that negative shocks to returns are likely to be associated to positive shocks to volatility: negative shocks to financial markets would trigger higher volatility and riskiness. Of course for $\rho = 0$, the model would be simply the regular SV-t model with no leverage effect.

(5) Standard Normal errors with leverage effect

$$
\begin{aligned}
y_t &= \mu + \sigma_t z_t \\
z_t &\sim N(0; 1) \\
\ln \sigma_t^2 = h_t &= \alpha + \phi\left(\ln \sigma_{t-1}^2 - \alpha\right) + \eta_t \\
\eta_t &= \rho z_t + \sqrt{1-\rho^2}\,\varepsilon_t \\
\varepsilon_t &\sim N\left(0; \sigma_\eta^2\right)
\end{aligned}
$$

where ρ is the coefficient driving the leverage effect in the SV-N-L model.

(6) Standard Asymmetric Laplace distributed errors with leverage effect

$$
\begin{aligned}
y_t &= \mu + \sigma_t z_t \\
z_t &\sim ALD(0; \kappa; 1) \\
\ln \sigma_t^2 = h_t &= \alpha + \phi\left(\ln \sigma_{t-1}^2 - \alpha\right) + \eta_t \\
\eta_t &= \rho z_t + \sqrt{1-\rho^2}\,\varepsilon_t \\
\varepsilon_t &\sim N\left(0; \sigma_\eta^2\right)
\end{aligned}
$$

where ρ is the coefficient driving the leverage effect in the SV-ALD-L model.

3. VaR and CVaR models

Considering $VaR_{s;t}(l)$ and $VaR_{d;t}(l)$ as the VaR for oil supply and demand in l-period with con dence level (1) 2 $(0;1)$ respectively, then, we have:

$$Supply: \quad Prob\,(y_t(l) \quad VaR_{s;t}(l) j \quad _t) = \qquad (3)$$

$$Demand: \quad Prob\,(y_t(l) \quad VaR_{d;t}(l) j \quad _t) = \qquad (4)$$

where $y_t(l)$ represents the oil return series for period (from t to $t + l$), $_t$ is the information set up to time t, is the risk level, and the value of $VaR_{s;t}$ and $VaR_{d;t}$ are de ned to be positive. Likewise, $CVaR_{s;t}(l)$ and $CVaR_{d;t}(l)$ are de ned as the CVaR of oil supply and demand respectively over period l at con dence level (1), and they can be mathematically expressed as:

$$Supply: \quad CVaR_{s;t}(l) = \quad E f y_t(l) j y_t(l) \quad VaR_{s;t}(l) g \qquad (5)$$

$$Demand: \quad CVaR_{d;t}(l) = E f y_t(l) j y_t(l) \quad VaR_{d;t}(l) g \qquad (6)$$

3.1. In the SV-N setting

Now we introduce the VaR and CVaR formulas under the SV-N framework.

Risk for oil Supply

(1) VaR: $VaR_{n;s;t} = \quad _t \quad ^{1}(\ )$

where $^{1}(\ )$ is the inverse cumulative distribution function of a N(0,1). In order to model the leverage e ect in this setting, we use $_t(\ )$.

(2) CVaR: $CVaR_{n;s;t} = \quad E \quad y_t j y_t \quad VaR_{n;s;t} \quad = \quad \frac{t}{} \ (\quad ^{1}(\ ))$

where $(\ )$ is the probability density function of a N (0,1). To model the leverage e ect in this setting, we use $_t(\ )$.

Risk for oil demand

(1) VaR: $VaR_{n;d;t} = \quad + \quad _t \quad ^{1}(\ )$

where $^{1}(\ )$ is the inverse cumulative distribution function of a N(0,1). To model the leverage e ect in this setting, we use $_t(\ )$.

(2) CVaR: $CVaR_{n;d;t} = E \quad y_t j y_t \quad VaR_{n;d;t} \quad = \quad + \frac{t}{} \ (\quad ^{1}(\ ))$

where ( ) is the probability density function of a N (0,1). To model the leverage e ect in this setting, we use $_t( )$.

3.2. *In the SV-ALD setting*

We now introduce the VaR and CVaR formulas under the SV-ALD model.

Risk for oil Supply

(1) VaR: $VaR_{s;t} = \quad + m_{s;q} \quad _t = \quad \frac{\sqrt{\ }_t}{\sqrt{2}} \ln \frac{(1 + \ ^2)}{2}$

where $m_{s;q} = (VaR_{s;t} + \quad)= \ _t$ is de ned as the left -quantile of the AL distribution. In order to model the leverage e ect in this setting, we use $_t( )$.

(2) CVaR: $CVaR_{s;t} = \quad E \quad y_t | y_t \quad VaR_{s;t} \quad = VaR_{s;t} + \frac{\sqrt{\ }_t}{\sqrt{2}}$

ToCVaR

4.1. Scale mixture of uniform representation of ALD

Expressing the ALD via the representation can alleviate the computational burden when using the Gibbs sampling algorithm in the MCMC approach and thus can simplify the estimation method in Bayesian analysis. To estimate the latent variables in the SV model, we use the scaled ALD (SALD) which means that the ALD random variable is scaled by its standard deviation (See Chen et al., 2009 and Wichitaksorn et al., 2015).

Proposition 1. *Let z_t be the ALD random variable with $z_t \quad ALD(0; \; ; 1)$, then the random variable $''_t = \frac{z_t}{S{:}D{:}[z]}$ has SALD with p:d:f: given by:*

$$f(''_t j \; ; \; _t) = \begin{cases} \frac{\sqrt{1+ \;^4}}{1+ \;^2} \frac{1}{\;_t} exp(\frac{\sqrt{1+\;^4}}{\;} \; ''_t) & ''_t \quad 0 \\ \frac{\sqrt{1+ \;^4}}{1+ \;^2} \frac{1}{\;_t} exp(\frac{\sqrt{1+\;^4}}{\;^2 \;_t} \; ''_t) & ''_t < 0 \end{cases} \tag{7}$$

where is skewness parameter and $_t$ is the standard deviation (or the time-varying volatility) of z_t.[5]

Hence, the corresponding SMU of SALD can be obtained as follows:

Proposition 2. *If $_t \quad Ga(2; 1)$ and $''_t \quad U(''_t j \quad \frac{^2 \; _t}{\sqrt{1+\;^4}}; + \frac{_t \; _t}{\sqrt{1+\;^4}})$, then the SMU density:*

$$f(''_t j \; ; \; _t; \; _t) = \int_0^1 f_U(''_t j \quad \frac{_t^2 \; _t}{\sqrt{1+\;^4}}; + \frac{_t \; _t}{\sqrt{1+\;^4}}) \quad f_{Ga}(\; _t j 2; 1) \, d \; _t \tag{8}$$

has the same form as the SALD density function given in equation (7).[6]

Using the SMU representation of SALD, an e cient simulation algorithm is developed to overcome parameter estimation di culties. As a result, the SV model discussed in section 2 can be written hierarchically as:

Return equation:

$$y_t j \; ; \; _t; h_t \quad U(\quad \frac{_t \; ^2 e^{h_t=2}}{\sqrt{1+\;^4}}; + \frac{_t e^{h_t=2}}{\sqrt{1+\;^4}}) \tag{9}$$

$$_t \quad Ga(2; 1) \tag{10}$$

[5]Note that original scale parameter has been canceled in this derivation, while the location parameter is set to be 0 in real practice. See appendix C for the derivation.

[6]See appendix D for the derivation.

Volatility equation:

$$h_t j \; ; \; ; \; ^2 ; h_{t \; 1} \quad N(\; + \; (h_{t \; 1} \quad); \; {}_2$$

ALD which, according to Kotz et al. (2001), is given by:

$$F(z| \; ; \; ; \;) = \begin{cases} 1 \quad \frac{1}{1 + \;^2} \, exp \left(\quad \frac{\sqrt{2}}{\quad} z \right) & z \quad 0 \\ \frac{\;^2}{1 + \;^2} \, exp \left(\quad \frac{\sqrt{2}}{\quad} z \; z \right) & \end{cases}$$

Table 1: MCMC estimation results for the SV-ALD model for the simulated data

Parameter	True	Mean	Median	SD	MC errors	95% CI
	-7.58700	-7.92400	-7.92300	0.90820	0.00647	(-9.48900, -6.35700)
	0.99470	0.99600	0.99610	0.00190	0.00006	(0.99160, 0.99910)
	0.08890	0.12880	0.125700	0.01610	0.00091	(0.10600, 0.17430)
	0.99560	0.97630	0.975100	0.01190	0.00068	(0.95610, 1.00200)

The following prior distribution are assumed: $N(\ \ 10; 0.001)$ with $0.001 = 1/\ ^2$; $Ga(2.5; 0; 025)$ with $\ = 1/\ ^2$; $Be(20; 1.5)$ with $\ = 1($

Table 2: Descriptive statistics for WTI and Brent oil price returns

	WTI	Brent
Panel A: Descriptive statistics		
Mean	-0.000144	-0.000127
Std.dev.	0.024863	0.021998
Maximum	0.164137	0.181297
Minimum	-0.128267	-0.168320
Skewness	0.1567	0.1443
Kurtosis	7.6122	8.8043
J-B test	2243.0570***	3547.5790***
Q(10)	30.6030***	16.9600*
Q(20)	60.8980***	54.2270***
ARCH(10)	475.9680***	215.7230***
ARCH(20)	575.8620***	409.0370***
Panel B: Unit roots and stationarity tests		
ADF	-51.4930***	-48.9570***
PP	-51.5220***	-48.9660***
KPSS	0.0507	0.0690

Note: Q(*l*) are Ljung--Box statistics for up to *lth*

Table 3: Posterior summary statistics for the parameters in SV-t, SV-N and SV-ALD models

Market	Parameter	Mean	SD	MC error	95% CI
		SV-t			
WTI		13.42000	2.83100	0.13450	(9.24600,20.10000)
		-9.77200	0.34470	0.00298	(-10.44000,-9.08500)
		0.99830	0.00101	0.00002	(0.99580,0.99970)
		0.09595	0.01143	0.00060	(0.07470,0.11840)
Brent		12.61000	2.50600	0.11710	(8.78900,18.43000)
		-9.76700	0.34190	0.00300	(-10.43000,-9.09200)
		0.99850	0.00094	0.00002	(0.99620,0.99980)
		0.08333	0.01086	0.00058	(0.06549,0.10730)
		SV-N			
WTI		0.00037	0.00034	0.00000	(-0.00030,0.00103)
		-9.76100	0.34660	0.00298	(-10.43000,-9.06800)
		0.99780	0.00122	0.00003	(0.99490,0.99950)
		0.11760	0.01360	0.00072	(0.09209,0.14630)
Brent		0.00013	0.00031	0.00000	(-0.00046,0.00074)
		-9.78700	0.34210	0.00277	(-10.45000,-9.10000)
		0.99860	0.00089	0.00002	(0.99640,0.99980)
		0.08933	0.01019	0.00054	(0.07112,0.11030)
		SV-ALD			
WTI		0.99560	0.01592	0.00083	(0.96590,1.02800)
		-7.58700	0.48730	0.00408	(-8.42200,-6.69800)
		0.99470	0.00224	0.00005	(0.98990,0.99870)
		0.08891	0.00992	0.00051	(0.07065,0.10810)
Brent		0.99820	0.01206	0.00061	(0.97510,1.02200)
		-7.75000	0.56200	0.00587	(-8.66000,-6.69600)
		0.99590	0.00184	0.00004	(0.99200,0.99910)
		0.07351	0.00812	0.00042	(0.06106,0.09410)

Table 4: Posterior summary statistics for the parameters in SV-t-L, SV-N-L and SV-ALD-L models

Market	Parameter	Mean	SD	MC error	95% CI
			SV-t-L		
		-0.62110	0.07616	0.00422	(-0.75940, -0.47680)
		10.90000	1.63300	0.06735	(8.22400, 14.50000)
WTI		-9.71700	0.35290	0.00376	(-10.39000, -9.01300)
		0.99830	0.00091	0.00002	(0.99610, 0.99960)
		0.09112	0.01027	0.00058	(0.07171, 0.10950)
		-0.57170	0.07303	0.00396	(-0.69400, -0.42250)
		12.38000	2.79900	0.13860	(8.56100, 19.59000)
Brent		-9.75300	0.34420	0.00327	(-10.42000, -9.07100)
		0.9986	0.00084	0.00002	(0.99650, 0.99970)
		0.08100	0.00946	0.00053	(0.06535, 0.10170)
			SV-N-L		
		-0.54850	0.07225	0.00387	(-0.66870,-0.39170)
		-0.00009	0.00035	0.00001	(-0.00078,0.00059)
WTI		-9.73200	0.35170	0.00310	(-10.41000,-9.02700)
		0.99810	0.00100	0.00002	(0.99570,0.99960)
		0.11110	0.01034	0.00057	(0.09117,0.13000)
		-0.62630	0.05649	0.00300	(-0.74130,-0.51490)
		-0.00025	0.00031	0.00000	(-0.00085,0.00035)
Brent		-9.78400	0.34170	0.00304	(-10.45000,-9.10800)
		0.99880	0.00070	0.00001	(0.99710,0.99980)
		0.08544	0.00821	0.00045	(0.07289,0.10570)
			SV-ALD-L		
		-0.74780	0.05345	0.00303	(-0.83640,-0.63140)
		1.00100	0.01336	0.00077	(0.97690,1.02700)
WTI		-7.75400	0.38370	0.00485	(-8.46500,-7.12300)
		0.99550	0.00156	0.00004	(0.99230,0.99840)
		0.09288	0.00826	0.00047	(0.07945,0.10980)
		-0.67460	0.06573	0.00369	(-0.78440,-0.53440)
		1.00700	0.01282	0.00073	(0.98060,1.02900)
Brent		-7.91800	0.53680	0.00447	(-8.93000,-6.97100)
		0.99690	0.00151	0.00005	(0.99340,0.99930)
		0.07427	0.00942	0.00055	(0.06148,0.09575)

Table 5: WTI: In sample Root Mean Square Error (RMSE)

Year	RMSE SV-t	RMSE SV-t-L	RMSE SV-N	RMSE SV-N-L	RMSE SV-ALD	RMSE SV-ALD-L
2006 2007	0.024666	0.024832	0.024440	0.024392	0.026936	0.026238

Table 8: Brent: In sample Mean Absolute Error (MAE)

Table 9: Out-of-sample performance for various models: RMSE and MAE for May-December 2016

Market	SV-t	SV-t-L	SV-N	SV-N-L	SV-ALD	SV-ALD-L
			RMSE			
WTI	0.034910	0.034952	0.033276	0.033010	0.038417	0.035692
Brent	0.038329	0.037781	0.038010	0.037333	0.037907	0.038116
			MAE			
WTI	0.026940	0.026973	0.025831	0.025362	0.028911	0.026794
Brent	0.029482	0.029095	0.029476	0.028731	0.028495	0.028391

the out-of-sample RMSE and MAE (to test the predictive power of the models) and, more importantly, the capability of the models to replicate risk in a VaR and CVaR sense.

Out-of-sample performance. Table 9 shows the out-of-sample performance for various models using the Root Mean square errors (RMSE) and Mean Absolute errors (MAE) criteria. We used the MCMC estimates from May 2006 to May 2016 to forecast oil returns from the end of May 2016 to the end of December 2016: the SV-N-L model performs better than its competitors for both markets if we consider the RMSE criterion (calculated using 500 simulations and fixing the parameters at the MCMC estimates). Considering the MAE criterion, SV-ALD-L and SV-N-L perform the best.

Table 10 to Table 15 present the results of Engle's LM ARCH test on the standard errors for SV-t, SV-t-L, SV-N, SV-N-L, SV-ALD and SV-ALD-L models in both markets. Considering the series of standard errors, there is no evidence of ARCH effects for the SV-N model while the SV-N-L model shows ARCH effects in the WTI market at a 1% significance level. This result gives an opportunity to increase efficiency by modeling ARCH, but does not violate any assumptions made when estimating the underlying model. As a conclusion, the SV-N model is the most efficient among the set of models that have been studied in

Table 10: WTI: Engle's Lagrange multiplier test for autoregressive conditional heteroskedasticity for standardised residuals and squared standardised residuals for SV-t and SV-t-L models

	1 lag	p-val	5 lags	p-val	10 lags	p-val	30 lags	p-val
SV-t res	2.04	0.15	8.98	0.11	13.75	0.18	47.15	0.02
SV-t res squ	0.19	0.66	0.82	0.98	1.66	1.00	11.50	1.00
SV-t-L res	3.51	0.06	9.46	0.09	12.79	0.24	39.48	0.12
SV-t-L res squ	0.09	0.76	0.57	0.99	0.98	1.00	31.60	0.39

Table 11: WTI: Engle's Lagrange multiplier test for autoregressive conditional heteroskedasticity for standardised residuals and squared standardised residuals for SV-N and SV-N-L models

	1 lag	p-val	5 lags	p-val	10 lags	p-val	30 lags	p-val
SV-N res	0.00	0.95	16.21	0.01	27.56	0.00	77.20	0.00
SV-N res squ	0.03	0.87	1.99	0.85	4.29	0.93	19.90	0.92
SV-N-L res	0.24	0.63	11.73	0.04	20.67	0.02	60.18	0.00
SV-N-L res squ	0.07	0.79	1.23	0.94	3.07	0.98	18.77	0.94

Table 12: WTI: Engle's Lagrange multiplier test for autoregressive conditional heteroskedasticity for standardised residuals and squared standardised residuals for SV-ALD and SV-ALD-L models

	1 lag	p-val	5 lags	p-val	10 lags	p-val	30 lags	p-val
SV-ALD res	7.31	0.01	9.72	0.08	11.20	0.34	35.24	0.23
SV-ALD res squ	0.63	0.43	1.04	0.96	1.49	1.00	13.45	1.00
SV-ALD-L res	13.69	0.00	15.75	0.01	18.05	0.05	39.41	0.12
SV-ALD-L res squ	7.55	0.01	8.13	0.15	8.62	0.57	36.25	0.20

Table 13: Brent: Engle's Lagrange multiplier test for autoregressive conditional heteroskedasticity for standardised residuals and squared standardised residuals for SV-t and SV-t-L models

	1 lag	p-val	5 lags	p-val	10 lags	p-val	30 lags	p-val
SV-t res	5.40	0.02	17.55	0.00	26.16	0.00	50.28	0.01
SV-t res squ	0.69	0.41	3.92	0.56	5.91	0.82	33.66	0.29
SV-t-L res	5.35	0.02	14.18	0.01	21.18	0.02	43.89	0.05
SV-t-L res squ	0.51	0.47	3.36	0.64	5.18	0.88	27.70	0.59

Table 14: Brent: Engle's Lagrange multiplier test for autoregressive conditional heteroskedasticity for standardised residuals and squared standardised residuals for SV-N and SV-N-L models

	1 lag	p-val	5 lags	p-val	10 lags	p-val	30 lags	p-val
SV-N res	7.17	0.01	22.48	0.00	34.65	0.00	70.86	0.00
SV-N res squ	1.22	0.27	6.91	0.23	10.47	0.40	31.61	0.39
SV-N-L res	5.38	0.02	13.79	0.02	21.96	0.02	47.77	0.02
SV-N-L res squ	0.78	0.38	4.29	0.51	7.01	0.72	22.95	0.82

Table 15: Brent: Engle's Lagrange multiplier test for autoregressive conditional heteroskedasticity for standardised residuals and squared standardised residuals for SV-ALD and SV-ALD-L models

	1 lag	p-val	5 lags	p-val	10 lags	p-val	30 lags	p-val
SV-ALD res	0.94	0.33	6.24	0.28	9.97	0.44	26.97	0.62
SV-ALD res squ	0.10	0.75	1.42	0.92	2.64	0.99	35.76	0.22
SV-ALD-L res	2.70	0.10	8.02	0.15	11.89	0.29	28.67	0.54
SV-ALD-L res squ	0.33	0.56	1.81	0.87	2.84	0.98	14.38	0.99

Table 16: WTI: Test Statistics and P-values for standardised residuals and squared standardised residuals for SV-t and SV-t-L models

	KSmirnov	p-val	SFrancia	p-val	Qtest	p-val
SV-t res	0.011	0.940	3.514	0.000	33.011	0.775
SV-t res squ	0.263	0.000	15.592	0.000	44.168	0.300
SV-t-L res	0.019	0.346	5.281	0.000	33.544	0.755
SV-t-L res squ	0.276	0.000	15.857	0.000	43.892	0.310

Table 17: WTI: Test Statistics and P-values for standardised residuals and squared standardised residuals for SV-N and SV-N-L models

	KSmirnov	p-val	SFrancia	p-val	Qtest	p-val
SV-N res	0.009	0.988	0.414	0.340	33.470	0.758
SV-N res squ	0.245	0.000	15.165	0.000	54.062	0.068
SV-N-L res	0.016	0.576	2.627	0.004	33.652	0.750
SV-N-L res squ	0.253	0.000	15.339	0.000	47.908	0.183

Table 18: WTI: Test Statistics and P-values for standardised residuals and squared standardised residuals for SV-ALD and SV-ALD-L models

	KSmirnov	p-val	SFrancia	p-val	Qtest	p-val
SV-ALD res	0.015	0.587	4.652	0.000	32.453	0.796
SV-ALD res squ	0.273	0.000	15.766	0.000	40.367	0.454
SV-ALD-L res	0.020	0.241	5.749	0.000	34.901	0.699
SV-ALD-L res squ	0.280	0.000	15.907	0.000	45.253	0.262

Table 19: Brent: Test Statistics and P-values for standardised residuals and squared standardised residuals for SV-t and SV-t-L models

	KSmirnov	p-val	SFrancia	p-val	Qtest	p-val
SV-t res	0.022	0.177	2.716	0.003	43.504	0.325
SV-t res squ	0.257	0.000	15.268	0.000	48.892	0.158
SV-t-L res	0.024	0.115	2.820	0.002	43.906	0.310
SV-t-L res squ	0.257	0.000	15.277	0.000	46.316	0.228

Table 20: Brent: Test Statistics and P-values for standardised residuals and squared standardised residuals for SV-N and SV-N-L models

	KSmirnov	p-val	SFrancia	p-val	Qtest	p-val
SV-N res	0.021	0.201	1.865	0.031	43.025	0.343
SV-N res squ	0.250	0.000	15.102	0.000	56.556	0.043
SV-N-L res	0.023	0.156	2.340	0.010	42.681	0.357
SV-N-L res squ	0.253	0.000	15.175	0.000	47.479	0.194

Table 21: Brent: Test Statistics and P-values for standardised residuals and squared standardised residuals for SV-ALD and SV-ALD-L models

	KSmirnov	p-val	SFrancia	p-val	Qtest	p-val
SV-ALD res	0.024	0.122	4.169	0.000	43.549	0.323
SV-ALD res squ	0.266	0.000	15.449	0.000	35.039	0.693
SV-ALD-L res	0.025	0.084	3.799	0.000	43.420	0.328
SV-ALD-L res squ	0.264	0.000	15.439	0.000	35.317	0.681

Table 22: Diebold Mariano test: comparison of forecast accuracy over 500 out-of-sample predictions

Variable	Observations	Mean	SD	Min	Max
		WTI			
SV-t vs SV-t-L					
r_{1t}	41	0.0010954	0.0004075	0.0006428	0.0023901
r_{2t}	41	0.0011213	0.0003617	0.0005928	0.0022897
SV-N vs SV-N-L					
r_{1t}	241	0.0012336	0.0005985	0.0005911	0.0046736
r_{2t}	241	0.0012112	0.0008895	0.0005486	0.0103819
SV-ALD vs SV-ALD-L					
r_{1t}	240	0.001657	0.0006006	0.0007806	0.0045232
r_{2t}	240	0.0012848	0.0008002	0.0006352	0.0051321
		Brent			
SV-t vs SV-t-L					
r_{1t}	46	0.0014817	0.0004475	0.0008498	0.0028845
r_{2t}	46	0.001352	0.000371	0.000789	0.0025136
SV-N vs SV-N-L					
r_{1t}	258	0.0015684	0.0006906	0.0007161	0.0063691
r_{2t}	258	0.0014911	0.0008635	0.0006984	0.0078067
SV-ALD vs SV-ALD-L					
r_{1t}	209	0.0015392	0.0006215	0.0007717	0.0041496
r_{2t}	209	0.00158	0.000905	0.0007069	0.0090912

and present summary statistics from that set of test results. Given an actual series and two competing predictions, one may apply a loss criterion (such as squared error or absolute error) and then calculate a number of measures of predictive accuracy that allow the null hypothesis of equal accuracy to be tested. Table 22 reports the results where the r_1 and r_2 variables are the MSEs for model 1 (*non-leverage model*) and model 2 (*leverage model*), respectively. If the $p \quad value < 0{:}05$, the test rejects the null that the two models are equally capable in terms of their MSEs. For the simulations in which the test rejects equal forecast accuracy, we can compare the mean MSE for the two models.

For the WTI data, in the case of SV-t vs SV-t-L models, we can observe 41 rejections (over 500 out-of-sample simulations): model 1 (the *non-leverage model*) has the smaller mean MSE. Considering SV-N vs SV-N-L models, we can observe 241 rejections: model 2 (*the leverage model*) has the smaller mean MSE. Considering SV-ALD vs SV-N-ALD models, we can observe 240 rejections: model 2 (*the leverage model*) has the smaller mean MSE. For the Brent data, in the case of SV-t vs SV-t-L models, we can observe 46 rejections: model 2 (*the leverage model*) has the smaller mean MSE. Considering SV-N vs SV-N-L models, we can observe 258 rejections: model 2 (*the leverage model*) has the smaller mean MSE. Considering SV-ALD vs SV-N-ALD models, we can observe 209 rejections: model 1 (the *non-leverage model*) has the smaller mean MSE.

In summary, in four of the six simulations, model 2 (*the leverage model*) has the smaller mean MSE for those simulations in which the Diebold{Mariano test rejects its null hypothesis of equal forecast accuracy.

7.3. Selection of VaR and CVaR models

We now focus on the models for which we have the most evidence of a substantial impact of the introduction of leverage on the prediction accuracy of the model (SV-N, SV-N-L, SV-ALD and SV-ALD-L models). In order to classify the competing models, we follow a two-stage model evaluation procedure where in the rst stage models are selected in terms of their statistical accuracy (*backtesting stage*), while in the second stage the surviving models are evaluated in terms of their e ciency (*e ciency stage*).[12]

Table 23: VaR backtesting results for WTI and Brent markets

	Risk	Failure times		Failure rate		LR_{uc}		LR_{ind}		LR_{cc}	
		WTI	Brent	WTI	Brent	WTI	Brent	WTI	Brent	WTI	Brent
					SV-N						
5%	VaR_{st}	107	122	4.248%	4.839%	0.0757	0.7099	0.8269	0.2114	0.1930	0.3869
	VaR_{dt}	111	116	4.407%	4.720%	0.1634	0.3754	0.9598	0.4968	0.3615	0.9944
1%	VaR_{st}	22	24	0.873%	0.952%	0.5138	0.8071	0.5334	0.5334	0.6598	0.7559
	VaR_{dt}	16	22	0.635%	0.873%	0.0486	0.5113	0.6510	0.6591	0.1282	0.6524
					SV-ALD						
5%	VaR_{st}	77	98	3.057%	3.887%	0.0000	0.0077	0.6766	0.9211	0.0000	0.0265
	VaR_{dt}	82	97	3.255%	3.848%	0.0000	0.0057	0.4314	0.2596	0.0001	0.0108
1%	VaR_{st}	7	8	0.278%	0.317%	0.0000	0.0001	0.8434	0.8214	0.0001	0.0003
	VaR_{dt}	5	6	0.198%	0.238%	0.0000	0.0000	0.8878	0.8656	0.0000	0.0000
					SV-N-L						
5%	VaR_{st}	112	126	4.446%	4.998%	0.1940	0.9964	0.3700	0.4934	0.2751	0.6781
	VaR_{dt}	103	109	4.086%	4.324%	0.0305	0.1111	0.3912	0.7538	0.0639	0.0105
1%	VaR_{st}	31	25	1.231%	0.992%	0.2615	0.9664	0.3793	0.4789	0.3572	0.7546
	VaR_{dt}	13	21	0.516%	0.833%	0.0071	0.3856	0.7134	0.5523	0.0249	0.5609
					SV-ALD-L						
5%	VaR_{st}	91	100	3.613%	3.967%	0.0008	0.0137	0.0210	0.5969	0.0002	0.0368
	VaR_{dt}	79	95	3.136%	3.768%	0.0000	0.0031	0.3571	0.8225	0.0000	0.0108
1%	VaR_{st}	9	6	0.357%	0.238%	0.0002	0.0000	0.7994	0.8656	0.0009	0.0000
	VaR_{dt}	5	5	0.198%	0.198%	0.0000	0.0000	0.8878	0.8878	0.0000	0.0000

Note: = 5% and 1% represent prescribed VaR level corresponding to 95% and 99% CI respectively, LR_{uc} columns show p-values of Kupiec's (1995) unconditional coverage test, LR_{ind} columns are p-values of Christo ersen's (1998) independent test and LR_{cc} columns are p-values of Christo ersen's (1998) conditional coverage test, * denotes signi cance.

Brent market especially when focusing on extreme tail risks (1%). Table 24 presents CVaR backtesting results for SV-N, SV-ALD, SV-N-L and SV-ALD-L models for oil supply and demand in the WTI and Brent markets. The performance of CVaR is very similar to the VaR performance. Looking at the p-values of the SV-N and SV-N-L model, they pass the three tests for the studied risk levels.

Considering both Tables 23 and 24, the main nding is that the introduction of the leverage e ect in the traditional SV model with Normally distributed errors is capable of adequately estimating risk (in a VaR and CVaR sense) for conservative (i.e. more risk averse, with = 5%) oil suppliers in both the WTI and Brent markets while it tends to overestimate risk for more speculative oil suppliers (= 5%). In comparison, the assumption of ALD errors

Table 25: RLF and FLF Loss function approach applied to the models surviving the VaR backtesting stage

Volatility models and VaR methods		RLF				FLF			
		5%		1%		5%		1%	
		WTI	Brent	WTI	Brent	WTI	Brent	WTI	Brent
Panel A: Average loss values									
SV-N	Supply	0.000209	0.000199	0.000170	0.000182	0.001709	0.001555	0.000328	0.002279
	Demand	0.000251	0.000237	0.000501	0.000227	-0.001680	-0.001531	-0.000499	-0.002274
SV-N-L	Supply	0.000239	0.000203	0.000176	0.000192	0.001755	0.001586	0.000353	0.002317
	Demand	0.000229	0.000219	0.000511	0.000162	-0.001733	-0.001569	-0.000511	-0.002313
SV-ALD	Supply	-	0.000250	-	-	-	0.001681	-	-
	Demand	-	-	-	-	-	-	-	-
SV-ALD-L	Supply	-	0.000235	-	-	-	0.001716	-	-
	Demand	-	-	-	-	-	-	-	-
Panel B: Sign statistics									
S_{AB}	Supply	47.2408	46.9433	49.1934	49.4129	-13.0107	-11.6517	-9.9422	-9.2612
	Demand	48.8746	49.0146	49.9904	49.9307	12.0144	10.6155	9.5438	8.8230
S_{BA}	Supply	48.4363	48.1382	49.9505	49.8909	13.0107	11.6517	9.9422	9.2612
	Demand	46.8822	46.5051	49.7115	49.5722	-12.0144	-10.6155	-9.5438	-8.8230
S_{CD}	Supply	-	48.1382	-	-	-	-7.1102	-	-
	Demand	-	-	-	-	-	-	-	-
S_{DC}	Supply	-	47.8594	-	-	-	7.1102	-	-
	Demand	-	-	-	-	-	-	-	-

Note: This table compares the best performing models in the VaR backtesting procedure following the Regulatory loss function (RLF) and Firm's loss function (FLF). Panel A presents the average loss values for RLF and FLF for the competing models at di erent risk levels in the two oil markets. The models with the lowest average loss values are underlined. Panel B reports the standardized sign statistics values. S_{AB} denotes the standardized sign statistics with null of \non-superiority" of SV-N over SV-N-L, S_{BA} represents the standardized sign statistics with null of \non-superiority" of SV-N-L over SV-N, S_{CD} is the standardized sign statistics with null hypothesis of \non-superiority" of SV-ALD over SV-ALD-L while S_{DC} is the standardized sign statistics with null hypothesis of \non-superiority" of SV-ALD-L over SV-ALD. * means signi cance in the corresponding level.

additional penalty related to the opportunity cost of capital.[13] We use a non-parametric sign test to check the ability the relevant VaR models to minimize these loss functions.[14]

Table 25 presents the summary results for the RLF and FLF loss function approach as applied to the models chosen in the VaR backtesting stage. The results in Panel A show that the SV-N model achieves the smallest value of average loss more often than the SV-N-L model while the outcome is not conclusive for the SV-ALD model and the SV-ALD-L model under the two approaches. To examine the statistical signi cance of the losses, we report the values of the standardized sign test in Panel B. Considering the RLF criterion, this test shows that the competing models (leverage vs no-leverage models) are not signi cantly di erent from

[13] This criterion penalizes large failures more than small failures (See Sarma et al., 2003).

[14] For the sign test see Lehmann (1974), Diebold and Mariano (1995), Hollander and Wolfe (1999) and Sarma et al. (2003).

Table 26 shows the summary results of RLF and FLF loss function approach applied to the models chosen in the CVaR backtesting stage. In terms of the average economic losses and considering both RLF and FLF as selection criteria, the SV-N model performs relatively better than the SV-N-L model in the WTI market while in the Brent market, the SV-N-L model outperforms the SV-N model. The standardized sign test values by FLF in the Panel B indicate that in most cases there are no signi cant di erences between the competitors. The only exception is that the SV-N-L model outperforms the SV-N model for oil supply in the Brent market at 1.96% risk level and the SV-ALD model performs better for oil demand in the WTI market at 0.37%

8. Conclusions

In this paper, we study the interaction between oil returns and volatility by using daily spot returns in the crude oil markets (both WTI and Brent) with a particular consideration for the impact of the leverage e ect on measures of risk such as VaR and CVaR. We nd that, allowing for leverage, traditional SV models with Normal distributed errors provide the best predictions in our out of sample experiments.

In order to address the risk faced by oil suppliers and oil consumers we model spot crude oil returns using Stochastic Volatility (SV) models with various error distributions. Among other cases, we test the assumption of Asymmetric Laplace Distributed (ALD) errors in order to model in a more distinctive way the type of risk faced by oil suppliers versus the risk faced by oil buyers.

We nd that the introduction of the leverage e ect in the traditional SV model with Normally distributed errors is capable of adequately estimating risk (in a VaR and CVaR sense) for conservative (i.e. more risk averse, with = 5%) oil suppliers in both the WTI and Brent markets while it tends apmdistributions.

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A. Asymmetric Laplace distribution

A random variable X is said to follow an Asymmetric Laplace Distribution if the characteristic function of X can be de ned as:

$$(t) = \frac{1}{1 + \frac{1}{2}\ {}^2 t^2 \quad i \ \ t} \tag{16}$$

where i is the imaginary unit, $t\ 2\ R$ is the argument of the characteristic function, is the scale parameter with > 0 and is the mean of X. Then, we have $X \quad AL(\ ;\ )$. Note that this characteristic function is a standardized form with location parameter $= 0$. An equivalent notation for the distribution of X can be written as $AL(\ ;\ )$. More details can refer to Kotz et al. (2001).

The density function is given by:

$$f(zj\ ;\ ;\ ) = \begin{cases} \frac{p\overline{2}}{} \frac{}{1 + \ {}^2} exp(\ \frac{p\overline{2}}{}(z \quad)) & z \\ \frac{p\overline{2}}{} \frac{}{1 + \ {}^2} exp(\frac{p\overline{2}}{}(z \quad)) & z < \end{cases} \tag{17}$$

B.

and further the CVaR for oil supply:

$$CVaR_{s;t} = E[y_t | y_t \quad VaR_{s;t}] = VaR_{s;t} + \frac{_t}{\sqrt{2}} \tag{19}$$

For oil demand, we have:

$$
\begin{aligned}
P(y_t > VaR_{d;t} | \quad _t) &= P\left(\frac{y_t}{_t} > \frac{VaR_{d;t}}{_t} \quad _t\right) \\
&= P\left(z_t > m_{d;q} = \frac{VaR_{d;t}}{_t}\right) = \int_{m_{d;q}}^{+\infty} f^{+}(z_t)\, dz_t \\
&= \int_{m_{d;q}}^{+\infty} \frac{\sqrt{2}}{} \frac{}{1 + \quad ^2} exp\left(\frac{\sqrt{2} \quad z_t}{}\right) dz_t \\
&= \quad \sqrt{}dz \quad m
\end{aligned}
$$

where = in our setting and is a constant, then we can transform z into another random variable $''_t$ by taking:

$$''_t = \quad z$$

D. Derivation of scaled ALD as an SMU

This part demonstrates the derivation of SALD as a scale mixture of $f_U(''_t j \quad \frac{p^{2}\ t}{1+\ ^4}; \quad + \frac{p\ t}{1+\ ^4})$ and $f_{Ga}(\ j2;1)$:

$$f(''_t j\ ;\ ;\ ;\ _t) = \int_0^1 f_U(''_t j \quad p\frac{^2\ _t}{1+\ ^4}; \quad + p\frac{_t}{1+\ ^4}) \quad f_{Ga}(\ j2;1)\,d \tag{27}$$

$$= \int_0^1 \frac{p\overline{1+\ ^4}}{1+\ ^2}\frac{1}{}\frac{1}{_t} I(\quad p\frac{^2\ _t}{1+\ ^4} < ''_t < \quad + p\frac{_t}{1+\ ^4}) \quad exp(\quad)\,d$$

Consider two cases for random variable $''_t$ where (1): $''_t > \quad \frac{p^2\ _t}{1+\ ^4}$ or equivalently $> \frac{p\overline{1+\ ^4}(''_t \quad)}{^2\ _t}$ and (2): $''_t < \quad + \frac{p\ _t}{1+\ ^4}$ or equivalently $> \frac{p\overline{1+\ ^4}(''_t \quad)}{_t}$.

Case (1):

$$\int_0^1 \frac{p\overline{1+\ ^4}}{1+\ ^2}\frac{1}{_t} I(\quad > \frac{p\overline{1+\ ^4}(''_t \quad)}{^2\ _t})\,exp(\quad)\,d$$

$$= \frac{p\overline{1+\ ^4}}{1+\ ^2}\frac{1}{_t}\int_{\frac{p\overline{1+\ ^4}(''_t\ )}{^2\ _t}}^1 exp(\quad)\,d(\quad) = \frac{p\overline{1+\ ^4}}{1+\ ^2}\frac{1}{_t} exp(\frac{p\overline{1+\ ^4}(''_t \quad)}{^2\ _t}) \tag{28}$$

Since $\frac{p\overline{1+\ ^4}(''_t \quad)}{^2\ _t} > 0$, thus we have $''_t <$, which follows:

$$f \quad (''_t j\ ;\ ;\ _t) = \frac{p\overline{1+\ ^4}}{1+\ ^2}\frac{1}{_t} exp(\quad p\overline{\quad}$$

i ; ;

As a result, it is demonstrated that the scaled Asymmetric Laplace density function of random variable ε_t:

$$f(\varepsilon_t | \ ; \ ; \ _t) = \begin{cases} \frac{\sqrt{1+\ ^4}}{1+\ ^2}\frac{1}{\ _t}\ exp(\frac{\sqrt{1+\ ^4}}{\ _t}(\varepsilon_t \quad)) & \varepsilon_t \\ \frac{\sqrt{1+\ ^4}}{1+\ ^2}\frac{1}{\ _t}\ exp(\frac{\sqrt{1+\ ^4}}{\ ^2\ _t}(\varepsilon_t \quad)) & \varepsilon_t < \end{cases} \tag{30}$$

can be replaced by an SMU distribution given by:

$$f(\varepsilon_t | \ ; \ ; \ ; \ _t) = \int_0^{\infty} f_U(\varepsilon_t | \quad \frac{\ ^2\ _t}{\sqrt{1+\ ^4}}; \quad + \frac{\ _t}{\sqrt{1+\ ^4}}) \quad f_{Ga}(\ |2;1)\, d \tag{31}$$

For latent variables h_t, we have:

$$
\begin{aligned}
f(h_t|h_{\ t};\ ;\ ;\ ^2;y) &\propto f(y|h_t;\ ;\ ;\ ^2)\,f(h_t|h_{\ t};\ ;\ ;\ ^2) \\
&= \frac{1}{\frac{e^{h_t/2}}{\sqrt{1+\ 4}} + \frac{\ ^2 e^{h_t/2}}{\sqrt{1+\ 4}}} \frac{1}{\sqrt{2\ B^2}} \exp\left(\ \frac{(h_t\ \ A)^2}{2B^2}\right) \\
&\propto e^{\ \frac{h_t}{2}} \exp\left(\ \frac{1}{2}\left(\frac{h_t^2}{B^2}\ \ 2h_t\frac{A}{B^2}\right)\right) \\
&= \exp\left[\ \frac{1}{2}\Bigg(\underbrace{\frac{1}{B^2}}_{C} h_t^2\ \ 2h_t \underbrace{\left(\frac{A}{B^2}\ \ \frac{1}{2}\right)}_{D}\Bigg)\right]
\end{aligned}
\tag{35}
$$

where

$A2$