AFFIRMATIVE ACTION IN INDIA VIA VERTICAL AND HORIZONTAL RESERVATIONS

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ABSTRACT. Built into the country's constitution, one of the world's most comprehensive affirmative action programs exists in India. Government jobs and seats at publicly funded educational institutions are allocated through a Supreme Court-mandated procedure that integrates a meritocracy-based system with a reservation system that provides a level playing field for disadvantaged groups through two types of special provisions. The higher-level provisions, known as vertical reservations, are exclusively intended for backward classes that faced historical discrimination, and implemented on a "set aside" basis. The lower-level provisions, known as horizontal reservations, are intended for other disadvantaged groups (such as women or disabled citizens), and they are implemented on a "minimum guarantee" basis. We show that, the Supreme Court-mandated procedure suffers from two major deficiencies: Not only a candidate can lose a position to a less meritorious candidate from a higher-privilege group, completely against the philosophy of affirmative action, but she can also lose a position simply because of disclosing her disadvantaged status. This loophole under the Supreme Court-mandated procedure causes widespread confusion in India, resulting in countless lawsuits, conflicting judgements on these lawsuits, and even defiance in some of its states. A recent amendment in the Constitution of India has a potential to amplify the adverse effects of these shortcomings, especially to the detriment of female candidates. We propose an alternative procedure that resolves both deficiencies with the smallest possible deviation from the Supreme Court-mandated procedure.

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1. Introduction

While the term "affirmative action" was first used in 1961 when President John F. Kennedy signed Executive Order 10925, the 1950 Constitution of India had already mandated affirmative action to the members of its so-called "backward classes." The intended groups were Scheduled Castes (SC), which is the official term for Dalits or "untouchables," whose members have suffered millenniums-long systematic injustice due to their lowest status under the caste system, and Scheduled Tribes (ST), which is the official term for the indigenous ethnic groups of India, whose members were both physically and socially isolated from the rest of the society. Built into the country's constitution, affirmative action has been implemented in India through a reservation system that earmarks a cer-

> satisfactorily through political processes. But that was not to be. The issues have been relegated to the judiciary...
>
> There are other reasons, of course - that cause governments to leave decisions to be made by Courts. They are of expedient political character. The community may be so divided on a particular issue that a government feels that the safe course for it to pursue is to leave the issue to be resolved by the Courts, thereby diminishing the risk it will alienate significant sections of the Community.

India is a federal union that consists of twenty-nine states with a unitary, three-tiered judiciary made up of lower trial courts, a high court for each state, and a Supreme Court above all courts. The Supreme Court is not only vested with original jurisdiction to issue writs in defence of the fundamental rights listed in the Constitution, but also with appellate jurisdiction from the high courts to review and change the outcomes of their decisions (Neuborne, 2003). As a result, the Supreme Court of India has always played a central role in matters of affirmative action.

In Indra Sawhney (1992), the Constitution bench of the Supreme Court formulated vertical reservations (also called social reservations) as a tool to implement the higher-level provisions enabled by Article 16(4), and horizontal reservations (also called special reservations) as a tool to implement the lower-level provisions enabled by Article 16(1). The scope and the mechanics of these two types of reservations were distinctly differentiated in this judgement as follows:

(1) Vertical reservations:
   (a) They are the highest form of special provisions that are intended exclusively for members of backward classes SC, ST, and OBC.
   (b) Being the highest form of special provisions, these reserved positions are to be earmarked to the members of backward classes in the form of a "set aside," which means positions secured by members of these classes on the basis of their own merit are not counted against vertically reserved positions.
   (c) They cannot exceed 50% of the positions.[2]

[2]Not all states follow the 50% upper bound for vertical reservations. Most notable example is Tamil Nadu with 69.5%. See The Print story "4 states have gone over SC-imposed 50 percent reservation cap.TJ/Frvations

(2) Horizontal reservations[3]
   (a) They are the lesser form of special provform ferol

To the best of our knowledge, our paper is the first one that formulates and analyses vertical and horizontal reservations, when both types of provisions coexist. From a market-design angle, we show that the reference procedure given in Anil Kumar Gupta (1995)and mandated throughout India has two important shortcomings. Our main contributions are:

(1) formulating these shortcomings in Section 4.1,
(2) documenting that they are responsible for numerous lawsuits throughout India in Section 6, and
(3) resolving them through an alternative procedure in Section 5.

We also relate these shortcomings to the One Hundred and Third Amendment of the Constitution of India in Section 7, and argue that the adverse impact of these shortcomings will likely increase considerably since the amendment interacts rather poorly with the vulnerabilities of the Supreme Court-mandated procedure.

While vertical and horizontal reservations are introduced to protect disadvantaged groups, the Supreme Court-mandated procedure allows for situations where a candidate from a disadvantaged group, despite being more meritorious, may still lose a position to a candidate from a more privileged group. We refer to this irregularity as a failure to eliminate justi ed envy. This failure is highly inconsistent with the principle of inter semerit built into the Constitution of India, whereby a candidate can never lose a position to a less meritorious candidate provided that they are from the same group. Indeed, under the Supreme Court-mandated procedure a candidate can never lose a position to a less meritorious candidate from the same group, but ironically she can lose a position to a less meritorious candidate from a more privileged group. In addition to this highly implausible possibility, the Supreme Court-mandated procedure may also penalize candidates for reporting their vertical reserve-eligible backward class. In that sense, the procedure is not incentive compatible. These two shortcomings not only result in countless lawsuits, but also provide a loophole in the procedure that can be used to discriminate against members of backward classes. In Section 6, we provide ample evidence that these shortcomings are responsible for widespread confusion in India, often resulting in legal action, and even defiance in some states through the illegal implementation of better-behaved versions of the mandated procedure. We also provide evidence in Section 6.2 that, in some jurisdictions these shortcomings are exploited by local officials to discriminate against members of backward classes. These litigations often result in the interruption of the recruitment process, as well as reversals of recruitment decisions. Reporting a judgement by the Gujarat High Court, an article in The Times of India highlights this very issue:[7]

[7]The Times of India story is available at https://timesofindia.indiatimes.com/city/ahmedabad/general-seat-vacated-by-quota-candidate-remains-general-hc/articleshowprint/57658109.cms (last accessed on 04/12/2019).

choice rule of Echenique and Yenmez. Ehlers et al. (2014) study more general affirmative action policies that adjust the priorities of students depending on the number of admitted students with different types.

More broadly, our paper contributes to market design, where economists are increasingly taking advantage of advances in technology to design new or improved allocation mechanisms in applications as diverse as entry-level labor markets (Roth and Peranson, 1999), school choice (Balinski and Sönmez, 1999; Abdulkadiroglu and Sönmez, 2003), spectrum auctions (Milgrom, 2000), kidney exchange (Roth et al., 2004, 2005), internet auctions (Edelman et al., 2007; Varian, 2007), course allocation (Sönmez and Ünver, 2010; Budish, 2011), cadet-branch matching (Sönmez and Switzer, 2013; Sönmez, 2013), assignment of arrival slots (Schummer and Vohra, 2013; Schummer and Abizada, 2017), refugee matching (Jones and Teytelboym, 2017; Delacrétaz et al., 2016; Andersson, 2017), and interdistrict school choice (Hafalir et al., 2018).

2. Institutional Background on Vertical and Horizontal Reservations

In Indra Sawhney (1992), the Constitution bench of the Supreme Court coined the terms vertical reservation and horizontal reservation, while emphasizing in the following statement how these two types of affirmative action tools are to interact with each other:

> A little clarification is in order at this juncture: all reservations are not of the same nature. There are two types of reservations, which may, for the sake of convenience, be referred to as `vertical reservations' and `horizontal reservations'. The reservation in favour of scheduled castes, scheduled tribes and other backward classes [under Article 16(4)] may be called vertical reservations whereas reservations in favour of physically handicapped [under clause (1) of Article 16] can be referred to as horizontal reservations. Horizontal reservations cut across the vertical reservations -- what is called interlocking reservations. To be more precise, suppose 3% of the vacancies are reserved in favour of physically handicapped persons; this would be a reservation relatable to clause (1) of Article 16. The persons selected against his quota will be placed in the appropriate category; if he belongs to SC category he will be placed in that quota by making necessary adjustments; similarly, if he belongs to opaare OC01) catego,ry he wild be placed in thad category by making necessary adjustmen.ns

vertical category including the open category (OC), the Supreme Court recommended the latter in their judgement of Anil Kumar Gupta (1995):

> We are of the opinion that in the interest of avoiding any complications and intractable problems, it would be better that in future the horizontal reservations are comparmentalised in the sense explained above. In other words, the notification inviting applications should itself state not only the percentage of horizontal reservation(s) but should also specify the number of seats reserved for them in each of the social reservation categories, viz., S.T., S.C., O.B.C. and O.C.

The compartment-wise implementation of horizontal reservations ensures that, unlike the aforementioned case, the distributional benefits of the special horizontal reservations extend to all segments of the society. Consistent with the Supreme Court's recommendation, many states in India have adopted compartment-wise implementation of horizontal reservations in their allocation of public positions. For example, in an effort to increase the participation of women in public employment, compartment-wise horizontal reservations for female candidates is mandated by government order in several states, including in Bihar with 35%, Andhra Pradesh with $33\frac{1}{3}$%, and Madhya Pradesh, Uttarak-

> then the process of verification and adjustment/accommodation as stated above should be applied separately to each of the vertical reservations.

The adjustment phase of the procedure for special horizontal reserves is further elaborated in the Supreme Court judgement Rajesh Kumar Daria v. Rajasthan Public Service Commission and others (2007) as follows:[11]

> If 19 posts are reserved for SCs (of which the quota for women is four), 19 SC candidates shall have to be first listed in accordance with merit, from out of the successful eligible candidates. If such list of 19 candidates contains four SC women candidates, then there is no need to disturb the list by including any further SC women candidate. On the other hand, if the list of 19 SC candidates contains only two woman candidates, then the next two SC woman candidates in accordance with merit, will have to be included in the list and corresponding number of candidates from the bottom of such list shall have to be deleted, so as to ensure that the final 19 selected SC candidates contain four women SC candidates. [But if the list of 19 SC candidates contains more than four women candidates, selected on own merit, all of them will continue in the list and there is no question of deleting the excess women candidate on the ground that 'SC-women' have been selected in excess of the prescribed internal quota of four.]

We refer to this choice rule as the *Supreme Court of India Vertical & Horizontal Reservations choice rule*, or *SCI-VHR choice rule* in short.

We are ready to present our formal model, theoretical results, and policy recommendations.

3. Model and Preliminary Results

Consider a finite set of individuals $\mathcal{I}$ who apply for q identical positions. Each individual either belongs to a reserve-eligible category such as "Scheduled Castes" (SC), "Scheduled Tribes" (ST), and "Other Backward Classes" (OBC), or belongs to the "General" category (G). For each reserve-eligible category, a number of positions is earmarked exclusively for the members of this category. In contrast, there are no positions earmarked for the members of the general category. Denote the set of reserve-eligible categories by $\mathcal{C}$.

Category membership is denoted by a function ρ. For an individual $i \in \mathcal{I}$ and a reserve-eligible category $c \in \mathcal{C}$, let $\rho(i) = \{c\}$ indicate that i is a member of the category c. For an individual i, let $\rho(i) = \emptyset$ indicate that i is a member of the general category. For every set of individuals $I \subseteq \mathcal{I}$, let I^G denote the subset of general-category individuals in I and $I^R \equiv I \setminus I^G$ denote the subset of individuals in I with a reserve-eligible category.

[11] The case is available at `https://indiankanoon.org/doc/698833/` (last accessed on 03/12/2019).

In addition to being a member of a category, each individual also has a (possibly empty) set of traits. Each trait represents a disadvantage in the society, and the government may provide individuals who have this trait with easier access to positions to level the playing field. The set of traits is finite and denoted by $\mathcal{T}$. The set of traits of individual i is denoted by $t(i) \subseteq \mathcal{T}$.

Finally, each individual has a distinct merit score, where the score of individual i is denoted as $s(i) \in \mathbb{R}_+$.

An **allocation problem** is given by a tuple $\langle \mathcal{I}, \mathcal{C}, \mathcal{T}, r, t, s, q \rangle$.

Given an allocation problem, a **choice rule** is a function C such that for any $I \subseteq \mathcal{I}$

$$C(I) \subseteq I \text{ with } |C(I)| \leq q.$$

In words, for a given number of positions, q, and a set of individuals I who are applying for the positions, the choice rule C produces a subset of individuals who are allocated these positions. In addition, the set of individuals in I who are not chosen by C is denoted by $R(I)$. The choice rule may also depend on the affirmative action policies, which we explain next.

3.1. Vertical Reservations Only. Affirmative action for the social categories is implemented by setting aside a number of positions for each category. These reservations are called *vertical* or *social*. For any category $c \in \mathcal{C}$, let r^c denote the number of positions set aside for individuals from category c. The remaining $r^o = q - \sum_c r^c$ positions are open for all individuals, and they are called open-category positions. When there are only vertical

3.2. Horizontal Reservations Only. With the Article 16(1) of the 1950 Constitution of India, disadvantaged individuals with certain traits are provided with some lower-level provisions referred to as horizontal or special reservations. These reservations provide a

(1) $C(I)$ satis es the horizontal reservations for I, and
(2) $C(I)$ dominates I' for any other set I' $\subseteq$ I that satis es the horizontal reservations for I.

We do not take any position on whether merit maximality is the most adequate way to determine the set of “most deserving” candidates. In Section 4, however, we present how

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3.3. Vertical and Horizontal Reservations. We are ready to introduce the model in its full generality, with both vertical and horizontal reservations. For a social category $c \in \mathcal{C}$, we refer to the positions vertically reserved for its members as category-c positions. Similarly, positions open for individuals from all categories are referred to as open-category positions. For any trait $t \in \mathcal{T}$ and social category $c \in \mathcal{C}$, let r_t^c denote the number of category-c positions horizontally reserved for trait-t individuals from category-c. In addition, let r_t^o denote the number of open-category positions horizontally reserved for trait-t individuals. These horizontal reservations are provided on a minimum guarantee basis.

For each social category, assume that the sum of horizontal reservations for this category is no more than the number of positions set aside for this category, i.e., for every category $c \in \mathcal{C}$, $\sum_{t \in \mathcal{T}} r_t^c \leq r^c$. An analogous inequality also holds for open positions, i.e., $\sum_{t \in \mathcal{T}} r_t^o \leq r^o$.

4. SCI-VHR Choice Rule & Its Shortcomings

Before we formally define the SCI-VHR choice rule presented in Section 2.1 and mandated throughout India, we show that it is not well-defined when an individual can have more than one trait. That is, its outcome is not uniquely defined, and more specifically it may depend on the details of the adjustment process unspecified under Anil Kumar Gupta (1995)

Case 1 (t_1 t_2 t_3 t_4) : Of the four individuals, a and b are the only ones who qualify choice rule when individuals have at most one trait.[16] Similarly, we make the same assumption for any result on the SCI-VHR choice rule.

For a set of individuals who are allocated category-c positions, say that trait-t is **oversaturated for c** if the number of trait-t individuals assigned to category-c positions is strictly more than r^c_t. Say that an individual i who is assigned a category-c position is **exposed** if either she does not have a trait or her unique trait $t(i)$ is oversaturated for c. In addition, we also use the same terminology for individuals who are allocated open-category positions.

SCI-VHR Choice Rule C^{SCI}

Step 0: Construct the set of open-category eligible individuals I_1 as the union of the set of individuals with r^o highest merit scores and the set of general-category individuals.

Step 1(i): Tentatively choose the individuals with the r^o highest merit scores for the open-category positions.

Step 1(ii): If all open-category horizontal reservations are satisfied for I_1, then proceed to Step 2(i). Otherwise, for each trait t with unsatisfied open-category reservations for I_1, replace

the lowest merit score exposed chosen individual who has an open-category position with

the highest merit score unchosen general-category trait-t individual.

Repeat Step 1(ii) until all open-category horizontal reservations are satisfied for I_1.

Step 2(i): For each social category $c \in C$, let I^c_2 denote the set of category-c individuals who are not chosen yet. Tentatively choose the individuals in I^c_2 with the r^c highest merit scores.

Step 2(ii): For each social category $c \in C$ and trait $t \in T$ such that trait-t reservations are not satisfied for I^c_2, replace

the lowest merit score exposed individual with a category-c position with

the highest merit score unchosen category-c trait-t individual.

Repeat Step 2(ii) until all category-c horizontal reservations are satisfied for I^c_2.

This process ends in finite time, because, there can only be a finite number of iterations at Steps 1(ii) and 2(ii), and a distinct individual is chosen at each iteration.

[16] While in some practical applications candidates are requested to apply for at most one trait of hori-

that there are two open-category positions and one SC position available. Only one open-category position is reserved for women. Suppose the individuals have the following ranking according to their merit scores:

$$s(m_1^G) > s(m_2^G) > s(m_3^{SC}) > s(w_2^{SC}) > s(w_1^G).$$

When all individuals apply, C^{SCI} works as follows. At Step 1(i), m_1^G and m_2^G are tentatively chosen for the open-category positions. The horizontal reservation for women is not satisfied because no woman is allocated a general-category position and there is a rejected general-category woman. Therefore, an adjustment is made at Step 1(ii) and m_2^G is replaced with w_1^G. At Step 2(i), m_3^{SC} is tentatively chosen for the SC position. Since there are no women reservations for SC, no adjustment is made. The set of chosen individuals is $\{m_1^G, w_1^G, m_3^{SC}\}$.

There are two fundamental issues here. The first one is that even though w_2^{SC} has a higher merit score than w_1^G, and w_2^{SC} has a reserve-eligible category while w_1^G does not, w_2^{SC} is rejected while w_1^G is chosen. Woman w_2^{SC} has envy towards w_1^G and her envy is justified because w_2^{SC} has the same horizontal trait as w_1^G, she has a reserve-eligible category while w_1^G does not, and her merit score is higher than that of w_1^G.

The second issue is that if w_2^{SC} does not declare her category SC, then she will be considered a general-category woman and she will be allocated an open-category position at Step 1(ii) because her merit score is higher than that of w_1^G. Therefore, w_2^{SC} has incentives to not declare her caste status and participate as a general-category individual.

We next formalize these two conceptual issues with the SCI-VHR choice rule. To this end, first consider the following basic fairness property:

Definition 2. A choice rule C respects inter se merit if, for every $I \subseteq \mathcal{I}$ and $i, j \in I$ with

(1) $r(i) = r(j)$,
(2) $t(i) = t(j)$, and
(3) $s(i) > s(j)$

$$i \in C(I) \implies j \in C(I).$$

A choice rule respects inter se merit, if an individual with a higher merit score never loses a position to a lower merit score individual with an identical category and set of traits. It is easy to see that the choice rule C^{SCI} respects inter se merit, a concept that is mandated by several Supreme Court judgements, and deeply interwoven into modern Indian legal thought.

Given the importance of inter se merit in India, one would expect that the following stronger (but even more plausible) principle would also be respected under a Supreme Court-mandated procedure that implements the provisions for positive discrimination.

Definition 3. A choice rule C eliminates justified envy if, for every $I \quad \mathcal{I}$ and $i, j \in I$ with

(1) $r(i) \quad r(j)$,
(2) $t(i) \quad t(j)$, and
(3) $s(i) \quad s(j)$

$$i \in C(I) \implies j \in C(I).$$

In other words, there is justified envy for a choice rule whenever there exist two individuals i and j such that

1. (1) either i and j have the same category, or i is a general-category individual (thus lacking any reserve-eligible category),
2. (2) j has any trait that i has,
3. (3) j has a higher merit score than i, and
4. (4) j is rejected from a set of individuals while i is chosen.

Observe that individual j is either from a more disadvantaged category than individual i, or belongs to a more disadvantaged group of citizens possessing additional horizontal traits; and yet she loses a position to individual i despite having a higher merit score. Clearly this is a highly implausible situation. As such, eliminating justified envy is even more important than respecting inter semerit, at least in the context of positive discrimination.

Indeed the following quote from the Supreme Court judgement Rajesh Kumar Daria (2007)indicates the importance of this fairness principle in India:

> For example, if there are 200 vacancies and 15% is the vertical reservation for SC and 30% is the horizontal reservation for women, the proper description of the number of posts reserved for SC, should be: ''For SC: 30 posts, of which 9 posts are for women''. We find that many a time this is wrongly described thus : ''For SC : 21 posts for men and 9 posts for women, in all 30 posts''. Obviously, there is, and there can be, no reservation category of 'male' or 'men'.

The last part of this quote indicates that an SC woman cannot loose a position to an SC man of lower merit score, presumably because his set of reservation-qualified categories plus traits is strictly included in hers. Elimination of justified envy is formalization of this principle.

If a choice rule eliminates justified envy, then it also respects inter semerit. But even though C^{SCI} respects inter semerit, Example 2 shows that it does not eliminate justified envy because w_2^{SC} is rejected while w_1^{G} is chosen when all five individuals apply.

The second issue is that it is against the philosophy of reservation policies that declaring your reserve-eligible category or traits has a potential to hurt you in the allocation process. Before introducing this concept, we define the following auxiliary notion.

An individual **withholds some of her reserve-eligible privileges** if she does not declare either her backward category membership (in case she belongs to one), or some of her traits (or both). For example, a SC individual with a disability can withhold some of her reserve-eligible privileges by not declaring her SC membership or her disability.

Definition 4. A choice rule C is *incentive compatible* if, for every $I \subseteq \mathcal{I}$ and $i \in I$, if i is chosen from I by withholding some of her reserve-eligible privileges, then i is also chosen by declaring all of her reserve-eligible privileges.[17]

Incentive compatibility states the following: No individual should be losing a position simply because of declaring all her reserve-eligible privileges (i.e backward class membership or traits).[18] Example 2 shows that C^{SCI} is not incentive compatible because if w_2^{SC} is treated as a general-category female candidate, then she will be chosen when all five candidates apply whereas she is not chosen when she is treated as a SC woman.

5. Better-Behaved Alternatives to SCI-VHR Choice Rule

In this section, we provide two modifications of the Supreme Court's choice rule. Each one is not only well-defined regardless of how many traits each individual has, but it also addresses the two fundamental shortcomings identified in Section 4.1. Of the two choice functions, a possible adoption of the first one relies on an "easy to observe" fix, whereas a possible adoption of the latter choice function results in the least "invasive" reform. In that sense, we believe the latter choice function is the more plausible one from a market-design perspective.

The first alternative choice rule is a simple modification of the Supreme Court's choice rule:

Choice Rule C_{2s}^{hor}

Step 1: Apply $C^{hor}(\cdot \mid r^o, (r^o_t)_{t\in T})$ to the set of all individuals to allocate the open-category positions.

Step 2: For each social category $c \in \mathcal{C}$, apply $C^{hor}(\cdot \mid r^c, (r^c_t)_{t\in T})$ to the remaining category-c individuals and allocate category-c positions to the chosen individuals.

Since the source of the complications of the Supreme Court mandated choice rule was "hidden" in Step 0 of C_{1h}^{SCI} (which restricts access to open-category horizontal adjustments to a subset of the individuals), a straightforward remedy can be obtained by simply deeming every individual eligible for these adjustments, essentially removing Step 0. Under this alternative choice function the single-category merit-maximal choice rule C^{hor}

[17] Incentive compatibility of a choice rule was first introduced in Aygün and Bó (2016) in the context of affirmative action in Brazilian college admissions.

[18] Incentive compatibility and elimination of justified envy closely relate to each other in the presence of some standard axioms on choice functions. See Appendix A for the formal results.

without a position and allocate the open-open category positions to the chosen individuals. Stop if the set of individuals without a position remains the same.

Step 2k**,** k > 1**:** For each category c 2 C, apply $C^{hor}(\, jr^c, (r^c_t)_{t2T} \,)$ to the set of category-c individuals who are not allocated an open-category position in Step 2k 1 and allocate category-c positions to the chosen individuals. Stop if the set of individuals without a position remains the same.

This choice rule terminates in finite time because for a category either (1) the set of chosen individuals remains the same at a step and the rule terminates or (2) a different set of individuals is chosen and more horizontally reserved seats are filled or (3) a different set of individuals is chosen and the chosen set is ranked strictly higher by the dominance relation than the set chosen at the previous step.

Observe that Steps 0-2 of the choice rule C^{hor}_{ite} coincides with Steps 0-2 of the choice rule C^{SCI}_{1h}, where the latter is equivalent to C^{SCI} by Proposition 3 when each individual has at most one trait. The following example illustrates the working of C^{hor}_{ite}.[19]

Example 4. Consider a set I of 11 individuals with three general-category men m^G_1, m^G_2, m^G_3, two general-category women w^G_1, w^G_2, two SC men m^{SC}_4, m^{SC}_5, and three SC women w^{SC}_4, w^{SC}_5, w^{SC}_6. Suppose these individuals have the following merit-score ranking:

$$s(m^G_1) > s(m^{SC}_4) > s(m^G_2) > s(m^G_3) > s(w^{SC}_3) > s(w^{SC}_4) > s(m^{SC}_5) > s(w^{SC}_5) > s(w^G_1) > s(w^G_2) > s(w^{SC}_6).$$

There are four open-category positions, three of which are reserved for women. In addition, there are two SC positions, one of which is reserved for women.

When all individuals apply C^{SCI} works as follows. At Step 0, the set of open-category eligible individuals is constructed as

$$I_{1C}$$

for the open-category positions, which is

$$\{m_1^G, m_4^{SC}, m_2^G, m_3^G, m_5^{SC}, w_5^{SC}, w_1^G, w_2^G, w_6^{SC}\}.$$

Choice rule $C^{hor}(\,|r^o, r^o_w)$ is applied to this set and $\{m_1^G, w_5^{SC}, w_1^G, w_2^G\}$ is tentatively chosen for the open-category positions. At Step 4, all SC individuals who are not assigned an open-category positions are considered for the SC positions: m_4^{SC}, w_3^{SC}, w_4^{SC}, m_5^{SC}, w_6^{SC}. Choice rule $C^{hor}(\,|r^{SC}, r^{SC}_w)$ is applied to this set and individuals in $\{m_4^{SC}, w_3^{SC}\}$ are tentatively assigned SC positions. At Step 5, all individuals except those who are tentatively assigned SC positions at Step 4 are considered for the open-category positions, which is

$$\{m_1^G, m_2^G, m_3^G, w_4^{SC}, m_5^{SC}, w_5^{SC}, w_1^G, w_2^G, w_6^{SC}\}.$$

Choice rule $C^{hor}(\,|r^o, r^o_w)$ is applied to this set and $\{m_1^G, w_4^{SC}, w_5^{SC}, w_1^G\}$ is tentatively chosen for the open-category positions. At Step 6, all SC individuals not assigned to an open-category position, m_4^{SC}, w_3^{SC}, m_5^{SC}, w_6^{SC}, are considered for the SC positions. Choice rule $C^{hor}(\,|r^{SC}, r^{SC}_w)$ is applied to this set and individuals in $\{m_4^{SC}, w_3^{SC}\}$ are tentatively assigned SC positions. Since the set of unassigned individuals does not change at Step 6, the algorithm terminates and, therefore,

$$C^{hor}_{ite}(I) = \{m_1^G, m_4^{SC}, w_3^{SC}, w_4^{SC}, w_5^{SC}, w_1^G\}.$$

Finally, we also consider choice rule C^{hor}_{2s}. At the first step, all individuals are considered for the open-category positions and $C^{hor}(I|r^o, r^o_w) = \{m_1^G, w_3^{SC}, w_4^{SC}, w_5^{SC}\}$ is selected. At Step 2, all unassigned SC individuals m_4^{SC}, m_5^{SC}, w_6^{SC} are considered for SC positions and $C^{hor}(\,|r^{SC}, r^{SC}_w)$ is applied to this set to select $\{m_4^{SC}, w_6^{SC}\}$ for the SC positions. Hence,

$$C^{hor}_{2s}(I) = \{m_1^G, m_4^{SC}, w_3^{SC}, w_4^{SC}, w_5^{SC}, w_6^{SC}\}.$$

We are ready to present our results for the two alternative choice functions. As promised, both of them escape the shortcomings of the Supreme Court-mandated choice rule presented in Section 4.1.

Proposition 4. Both C^{hor}_{2s} and C^{hor}_{ite} eliminate justified envy and are incentive compatible.

We next show that between the choice rule C^{SCI} and its two alternatives C^{hor}_{2s}, C^{hor}_{ite},

(1) C^{SCI} produces the best and C^{hor}_{2s} produces the worst outcome for general-category individuals, whereas
(2) in terms of the total number of positions assigned to members of social categories, C^{hor}_{2s} produces the best outcome and C^{SCI} produces the worst outcome.

Proposition 5. Suppose that each individual has at most one trait. For every I,

$$C^{SCI}(I) \backslash \ I^G \quad C^{hor}_{ite}(I) \backslash \ I^G \quad C^{hor}_{2s}(I) \backslash \ I^G$$

and

$$C^{SCI}(I) \backslash \ I^R \quad C^{hor}_{ite}(I) \backslash \ I^R \quad C^{hor}_{2s}(I) \backslash \ I^R \ .$$

Proposition 5 shows that, of the two alternative choice function, the outcome of C^{hor}_{ite} is “closer” to the outcome of C^{SCI}

The algorithm terminates at Step 4 and C^{hor}_{ite} produces $f\, w^{SC}_3, d^{SC}_2, w^{SC}_2 g$. Therefore, $C^{SCI}(I) \; n \, C^{hor}_{ite}(I) = f\, w^{G}_1, d^{G}_1 g = 2$.

the petitioners seek legal action on the basis that reserve category women are allowed to bene t from open-category horizontally reserved positions for women. The high court rules that the state is at fault, and it must abandon its choice rule, adopting the one mandated by the Supreme Court. The following quote is from a story published in The Times of India covering this court case: [21]

> In a judgment that would affect all recruitments in the state government, the Rajasthan high court has ruled that posts reserved for women in the open/general category cannot be filled with women from reserved categories even if the latter are placed higher on the merit list ...
>
> Women candidates who contested for different positions in at least three government departments, including the panchayati raj, education and medical, last year had challenged the government move to allow ``migration'' of reserved category women to fill the open category seats. The positions applied for included that of teachers Grade-II and III, school lecturers, headmasters and pharmacists.

Ironically, while the high court's decision is correct, it also means that the better-behaved version of the choice rule has to be abandoned throughout the state.

(2) Ashish Kumar Pandey And 24 Others vs State Of U.P. And 29 Others on 16 March, 2016, Allahabad High Court. [22] In a case that mimics the aforementioned Rajasthan High Court case, this lawsuit was brought to Allahabad High Court by 25 petitioners, disputing the mechanism employed by the State of Uttar Pradesh—the most populous state in India with more than 200 million residents—to apply the provisions of horizontal reservations in their allocation of more than 4000 civil police and platoon commander positions. Of these positions, 27%, 21%, 2% are each vertically reserved for backward classes OBC, SC, and ST, respectively, and 20%, 5%, and 2% are each horizontally reserved for women, ex-servicemen, and dependents of freedom ghters, respectively. While only 19 women are selected for open-category positions based on their merit scores, the total number of female candidates is less than even the number of open-category horizontally reserved positions for women, and as such all remaining women are selected. However, instead of assigning them positions from their respective backward class categories

their Supreme Court-mandated form. While this resistance most likely reflects the political nature of this debate, the arguments of the counsel for the state to maintain their preferred mechanism to implement the provisions of horizontal reservations are mostly based on the presence of justified envy under the Supreme Court-mandated version. The following quote from the appeal illustrates that this was the main argument used in their defense:

> The arguments that have been advanced on behalf of State and private appellant with all vehemence that women candidates irrespective of their social class i.e. SC/ST/OBC are entitled to make place for themselves in an open category on their inter-se merit clearly gives an impression to us that State of U.P and its agents/servants and even the private appellants are totally unaware of the distinction that has been time and again reiterated in between vertical reservation and horizontal reservation and the way and manner in which the provision has to be pressed and brought into play.

(3) Asha Ramnath Gholap vs President, District Selection Committee & Ors. on March 3rd, 2016, Bombay High Court.[24] In this case, there are 23 pharmacist positions to be allocated; 13 of these positions are vertically reserved for backward classes and the remaining ten are open for all candidates. In the open category, eight of the ten positions are horizontally reserved for various groups, including three for women. The petitioner, Asha Ramnath Gholap, is an SC woman, and while there is one vertically reserved position for SC candidates, there is no horizontally reserved position for SC women. Under the SCI-VHR choice rule, she is not eligible for any of the horizontally reserved women positions at the open category. Nevertheless, she brings her case to the Bombay High Court based on an instance of justified envy, described in the court records as follows:

> It is the contention of the petitioner that Respondent Nos. 4 & 5 have received less marks than the petitioner and as such, both were not liable to be selected. The petitioner has, therefore, approached this court by invoking the writ jurisdiction of this court under Article 226 of the Constitution of India, seeking quashment of the select list to the extent it contains the names of Respondent Nos.4 and 5 against the seats reserved for the candidates belonging to open female category.

There is no merit to this argument, because the choice rule mandated by the Supreme Court allows for justified envy. However, the judges sided with the petitioner on the basis that a candidate cannot be denied a position from the open

[24] The case is available at `https://indiankanoon.org/doc/178693513/` (last accessed on 03/08/2019).

category based on her backward class membership, essentially ruling out the possibility of justified envy under a Supreme Court-mandated choice rule, which is designed to allow for positive discrimination for the vulnerable groups in the society.[25] Their justification is given in the court records as follows:

> We find the argument advanced as above to be fallacious. Once it is held that general category or open category takes in its sweep all candidates belonging to all categories irrespective of their caste, class or community or tribe, it is irrelevant whether the reservation provided is vertical or horizontal. There cannot be two interpretations of the words 'open category' ...

(4) Uday Sisode vs Home Department (Police) on 24 October, 2017, Madhya Pradesh High Court.[26] In another case parallel to that at Bombay High Court, the judges of Madhya Pradesh High Court issued a questionable decision by siding with a petitioner

However, the three judges side with the earlier judgement, thus erroneously dismissing the appeal. Their decision is justi ed as follows:

> The outstanding and important feature to be noticed is that it is not the case of the appellant-petitioner that she has obtained more marks than those 8 OBC (Woman) candidates, who have been appointed against the posts meant for General Category (Woman), inasmuch as, while the appellant is at Serial No.184 in the merit list, the last OBC (Woman) appointed is at Serial No.125 in the merit list. The controversy raised by the appellant is required to be examined in the context and backdrop of these significant factual aspects.

As seen from this argument, many judges have dif culty perceiving that the Supreme Court-mandated procedure could possibly allow for justi ed envy.

(6) Mukta Purohit & Ors vs State & Ors on 12 April, 2018, Rajasthan High Court.[28] In a case that mimics Smt. Megha Shetty (2013), judges of the Rajasthan High Court erroneously dismiss a petition led against the state that allowed horizontally reserved open-category women positions to be allocated to women from reserved categories who are not eligible for these positions. Indeed Smt. Megha Shetty (2013) is used as a precedent in this judgement.

(7) Arpita Sahu vs The State Of Madhya Pradesh on 21 August, 2012, Madhya Pradesh High Court. [29] The petitioner les a lawsuit based on an instance of justi ed envy, however in contrastoftoansidShetty6-309((2013))]TJ/F795Cou-76noftoShetty7-309((2013))]TJ/

(6)

merit score is higher than that of the lowest score candidate admitted for one of these positions, and that candidate, having the highest merit score among remaining Uttaranchal-women candidates, has to receive the horizontally reserved position Neetu Joshi no longer needs to occupy. The high court allows her petition, and in its decision grants her a position based on the following justification:

> In view of above, Neetu Joshi, (SI. No. 9, Roll No. 12320) has wrongly been counted by respondent No. 3 / Commission against five seats reserved for Uttaranchal Women General Category as she has competed on her own merit as general candidates and as 5th candidate the petitioner should have been counted for Uttaranchal Women General Category seats.

This erroneous high court judgement was later overruled by the Supreme Court in their civil appellate case Public Service ... vs Mamta Bisht And Ors on 3 June, 2010[33] but not before setting a precedent for several subsequent lawsuits.

6.2. Wrongful Implementation and Possible Misconduct. It is bad enough that the Supreme Court-mandated choice rule is not incentive compatible, forcing some candidates to choose between declaring their social reservation-eligible backward class status and their special reservation-eligible horizontal traits. To make matters worse, in some cases candidates are denied access to open-category horizontally reserved positions even when they do not submit their backward class status, giving up their eligibility for vertically reserved positions for their reserve-eligible class. Therefore, even when the candidate applies for a position as a general-category candidate, the central planner processes the application as if the backward class status was claimed, denying the candidate's eligibility for open-category horizontally reserved positions for her trait. The central planners are able to do this, because last names in India are, to a large extent, indicative of a caste membership. This type of misconduct seems to be fairly widespread, and it is the main cause of the lawsuit in each of the following cases:

(1) Shilpa Sahebrao Kadam And Another vs The State Of Maharashtra And ... on 8 August, 2019, Bombay High Court.[34]
(2) Vinod Kadubal Rathod And Another vs Maharashtra State Electricity ... on 17 February, 2017, Bombay High Court.[35]
(3) Original Applications 1007, 1052, 1056, 1057 & 1070/2017 dated 29.11.2017, Maharashtra Administrative Tribunal, Mumbai Bench.[36]

[33]The case is available at https://indiankanoon.org/doc/518824/ (last accessed on 03/07/2019).
[34]The case is available at https://indiankanoon.org/doc/89017459/ (last accessed on 03/09/2019).
[35]The case is available at https://indiankanoon.org/doc/162611497/ (last accessed on 03/09/2019).
[36]The case is available at https://mat.maharashtra.gov.in/Site/Upload/Pdf/O.A%201007.17%20and%20ors%20DB,%2029.11.17,%20Chairman.PDF (last accessed on 03/09/2019).

(4)

> Another issue relates to the access of SCs and STs to the institutions of justice in seeking protection against discrimination. Studies indicate that SCs and STs are generally faced with insurmountable obstacles in their efforts to seek justice in the event of discrimination. The official statistics and primary survey data bring out this character of justice institutions. The data on Civil Rights cases, for example, shows that only 1.6% of the total cases registered in 1991 were convicted, and that this had fallen to 0.9% in 2000.

6.3. Loss of Access to Horizontal Reservations without any Access to Vertical Reservations. The main justification offered in various Supreme Court cases for denying backward class members the provisions of horizontal reservations for open-category positions is avoiding a situation where an excessive number of positions are reserved for members of these classes. In several cases, however, members of these classes are denied access to horizontally reserved positions even when their reserve-eligible vertical category is not earmarked for those positions. This is the case in the following two court cases:

(1) Tejaswini Raghunath Galande v. The Chairman, Maharashtra Public Service Commission and Ors. on 23 January 2019, Writ Petition Nos. 5397 of 2016 & 5396 of 2016, High Court of Judicature at Bombay.[40]
(2) Original Application No. 662/2016 dated 05.12.2017, Maharashtra Administrative Tribunal, Mumbai.[41]

In both of the above cases, while both petitioners declared their backward class status, there was no position vertically reserved for their class. Yet they both lost access to horizontally reserved positions in the open category for their traits. In the first case, the petitioners' lawsuit to benefit from horizontal reservations was initially declined by a lower court, resulting in the appeal at the High Court. The lower court's decision was overruled in the High Court, and her request was granted. The second petitioner's similar request was declined by the Maharashtra Administrative Tribunal. What is more worrisome in the second case is that initially three positions were announced to be vertically reserved for the petitioner's backward class, but after her application these positions were withdrawn. Therefore, the candidate declared her backward class status, giving up her eligibility for several horizontally reserved women positions at the open category, presumably to gain access to vertically reserved positions for her backwards class, only to learn that she had given up her eligibility for nothing.

[40]The case is available at https://www.casemine.com/judgement/in/5c713d919eff4312dfbb5900 (last accessed on 03/09/2019).

[41]The case is available at https://mat.maharashtra.gov.in/Site/Upload/Pdf/O.A.662%20of%202016.pdf (last accessed on 03/09/2019).

7. The Implications of 103rd Amendment of the Constitution of India

In a highly debated reform on the reservation system, the One Hundred and Third Amendment of the Constitution of India provides 10% reservation to the economically weaker sections (EWS) in the general category.[42] A government memorandum dated 01/31/2019 specifies these new provisions as a vertical reservation:[43]

> 7. ADJUSTMENT AGAINST UNRESERVED VACANCIES:
>
> A person belonging to EWS cannot be denied the right to compete for appointment against an unreserved vacancy. Persons belonging to EWS who are selected on the basis of merit and not on account of reservation are not to be counted towards the quota meant for reservation.

The One Hundred and Third Amendment was immediately challenged at the Supreme Court, and as of October 2019 the case is still pending.[44] Despite being challenged at the Supreme Court, the EWS reservation has already been adopted by federal institutions throughout India as well as by most states at their state-run public institutions. If the One Hundred and Third Amendment survives the Supreme Court challenge, it will likely amplify the legal challenges formalized in Section 4.1 and documented in Section 6. Especially in states with a strong presence of horizontal reservation (such as those with 30-35% women reservation), legal challenges based on justified envy may become the norm rather than an exception if the amendment survives. That is because, any candidate who applies both for the EWS reservation and any horizontal reservation will loose access to open-category horizontally reserved positions under the Supreme e81m of this group satisfy the eligibility criteria for the EWS reservation.[46] This means, with the introduction of the EWS reservation, the fraction of the population who are ineligible for any vertical reservation reduces to roughly 5-6% of the population. Therefore, the "new general category," those members of the society who are ineligible for any vertical reservations, shrinks to approximately 5-6% of the whole population.[47] A key implication of this observation is the following: Unless the Supreme Court-mandated choice rule is amended in a manner addressing the shortcomings presented in Section 4.1, only this "elite" 5-6% of the population qualifies for adjustments for open-category horizontal reservation. For example, consider a woman who qualifies for the 10% EWS reservation. In a state with 30% women reservation, she will now be qualified for the horizontally reserved EWS-women positions which makes 3% of all positions. However, on the other hand she will loose access to open positions that are horizontally reserved for women which is 12% of all positions. This anomaly will likely increase the instances of justified envy considerably throughout India, especially in states with extensive use of horizontal reservation, such as Bihar with 35%, Andhra Pradesh with $33\frac{1}{3}$%, and Madhya Pradesh,

(2) SC/ST and OBC cannot be excluded from economic reservations, as this would violate the fundamental right to equality.
(3) The Amendment introduces reservations that exceed the 50% ceiling-limit on reservations, established by Indra Sawhney.
(4) Imposing reservations on educational institutions that do not receive state aid violates the fundamental right to equality.

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Appendix A. The Relation Betwee223Matc Tf -422.2o [(,e9oMptioblita783 Td66 -2.57 Td [(Un9.e2

Proposition 6. Suppose that choice rule C is incentive compatible, respects inter se merit, and satisfies the irrelevance of rejected individuals. Then it eliminates justified envy.

Next we introduce another property that is also studied in a two-sided matching context (Kelso and Crawford, 1982).

Definition 6. Choice rule C satisfies *substitutability* if for every $I \subseteq \mathcal{I}$, $i \in C(I)$, and $j \neq i$, we have $i \in C(I \setminus \{j\})$.

Substitutability states that an individual chosen from a set of applicants is still chosen if other individuals are removed from the set.

Proposition 7. Suppose that choice rule C eliminates justified envy, satisfies substitutability and the irrelevance of rejected individuals. Then it is incentive compatible.

In the next example, we provide a choice rule that eliminates justified envy and satisfies the irrelevance of rejected individuals but is not incentive compatible.

Example 6. Consider the following choice rule. If there exist at least two general-category individuals, choose two of them who have the highest merit scores. In this case, let the

Consider all individuals in I who have at least one trait with a reserved position, say $\tilde{I}_1$. If $\tilde{I}_1$ is empty, then choose one individual in I. Otherwise, choose one individual from $\tilde{I}_1$ and decrease the number of reserved positions for the traits of this individual by one. Consider the set of remaining individuals in $\tilde{I}_1$ who have at least one trait with a reserved position, say $\tilde{I}_2$. If $\tilde{I}_2$ is empty, then choose one of the remaining individuals from I. Otherwise, if $\tilde{I}_2$ is not empty, then choose an individual from $\tilde{I}_2$. Continue this procedure so that the number of chosen individuals is $\min\{q, |I|\}$. We claim that the chosen subset, say I', satisfies the horizontal reservations for I. Suppose, for contradiction, that it does not. Then there exists a trait t such that the number of individuals with trait t in I' is less than r_t and that there is at least one individual in $I \setminus I'$ with trait t. In this case, $|I'| = q$ because $I \setminus I'$ is nonempty. Since the number of remaining reserved positions for trait t is positive, and an individual with this trait is not chosen at the last step, an individual with a trait that has a positive reserved position is chosen at every step. But this is a contradiction to the assumption that $\sum_{t \in T} r_t \leq q$. Therefore, there exists at least one subset of I that satisfies the horizontal reservations for I.

Let $I' = \{i'_1, \ldots, i'_n\}$ be a subset of I that satisfies the horizontal reservations for I and $C^{hor}(I) = \{i_1, \ldots, i_m\}$. Suppose that $I' \neq C^{hor}(I)$. Re-order individuals in each set so that individuals with a lower index have higher merit scores than individuals with a higher index. We claim that $C^{hor}(I)$ dominates I'. Let k be the minimum index such that $i_k \neq i'_k$. By construction of $C^{hor}(I)$, i_k has a higher merit score than all individuals in $I' \setminus C^{hor}(I)$ because at Step k both $C^{hor}(I)$ and I' are considered and individual i_k is chosen by C^{hor}. Therefore, $C^{hor}(I)$ dominates I'.

Proof of Proposition 2. Before we start the proof, we introduce some notation. For any set of individuals $I \subseteq \mathcal{I}$ and trait $t \in T$, let $I_t \equiv \{i \in I | t \in \tau(i)\}$. In words, I_t is the set of individuals in I who have trait t. We use the following lemma in the proof.

Lemma 1. $\bar{I} \subseteq I$ satisfies the horizontal reservations for I if, and only if, $|\bar{I}_t| \geq \min\{r_t, |I_t|\}$ for every trait $t \in T$.

Proof. First we show sufficiency. Let $\bar{I}$ be such that $|\bar{I}_t| \geq \min\{r_t, |I_t|\}$ $|\bar{}$

the horizontal reservations for I. Let I^0 be the set of individuals chosen out of I by C^{mg} in Step 1. $C^{hor}(I)$ must include all the individuals in I^0 because by Lemma 1 the number of trait-t individuals in $C^{hor}(I)$ is at least $\min\{r_t, |I_t|\}$. Furthermore, by the construction of C^{hor}, whenever a trait-t individual is chosen, it always selects the trait-t individual with the highest merit score from the available set, so $C^{hor}(I)$ I^0.

Now, if $C^{mg}(I) \cap I^0 \neq C^{hor}(I) \cap I^0$, then $C^{mg}(I) \cap I^0$ would dominate $C^{hor}(I) \cap I^0$ by the construction of C^{mg} because it selects individuals with the highest merit score in Step 2. Therefore, $C^{mg}(I)$ would dominate $C^{hor}(I)$ because adding or subtracting a set of individuals preserves the domination relationship. But this cannot hold because C^{hor} is merit maximal and so $C^{hor}(I)$ dominates any subset of I different from $C^{hor}(I)$ that satisfies the horizontal reservations for I. Therefore, $C^{mg}(I) \cap I^0 = C^{hor}(I) \cap I^0$, and thus $C^{mg}(I) = C^{hor}(I)$.

Proof of Proposition 3. Denote the union of the set of individuals with highest r^o merit scores and the set of all general-category individuals by I_1. In Step 1 of C^{SCI}_{1h}, $C^{hor}(I_1|r^o, (r^o_t)_{t \in T})$ is chosen for the open-category positions. We first show that the set of individuals chosen for the open-category positions by C^{SCI} is the same set.

When the open-category positions are allocated according to C^{SCI} in Steps 1(i) and 1(iiI se (r^o_t)9(on32.4573 Tf 4.77 0 Td 73 Tf 7.328.4573 Tf 7o 11.9552 Tf -$_{t \in T}$)375(individuals)-376(wit)1(h construction of C^{hor}, i cannot be chosen if i' is rejected. Therefore, C^{hor}_{2s} eliminates justified envy.

To show incentive compatibility of C^{hor}_{2s}, consider a set of individuals I and an individual $i \in I$ such that $i \notin C^{hor}_{2s}(I)$. Fix every other individual's category and set of traits. First note that C^{hor} does not use the categories of individuals, so modifying the category of i from a reserve-eligible category to general can only hurt i, as he will only be considered at the first step. Furthermore, declaring a set of traits $t \quad t(i)$ instead of $t(i)$ can only make this individual worse off, because if he is considered with set of traits t to satisfy some constraints, then he will also be considered with set of traits $t(i)$ to satisfy the same constraints. Therefore, C^{hor}_{2s} is incentive compatible.

To show elimination of justified envy of C^{hor}_{ite}, consider a set of individuals I and two individuals $i, i' \in I$ with with $r(i) \quad r(i')$, $t(i) \quad t(i')$, and $s(i) < s(i')$. At every Step k, where $k \quad 3$, when i is considered by C^{hor}_{2s}, i' is also considered. Furthermore, by the construction of C^{hor}, which is used at every step of C^{hor}_{2s}, an individual with a lower merit score and set of traits t is never chosen before another individual with a higher merit score and set of traits t' where $t' \quad t$. Since C^{hor}_{ite} terminates at Step 3 or later, By path independence of C^{hor}, $C^{hor}_{ite}(I) \cap I^G = C^{hor}(I_2) \cap I^G$ where I_2 is the set of individuals considered for the open-category positions at any step of C^{hor}_{ite}. Since $I_2 \subseteq I_1$ and $C^{hor}(I_2) \cap I^G \subseteq I_1$, by substitutability of C^{hor}, $C^{hor}(I_1) \cap I^G \supseteq C^{hor}(I_2) \cap I^G$, which is equivalent to $C^{SCI}(I) \cap I^G \supseteq C^{hor}_{ite}(I) \cap I^G$. Similarly, since $I \supseteq I_2$ and $C^{hor}(I) \cap I^G \subseteq I_2$, by substitutability of C^{hor}, $C^{hor}(I_2) \cap I^G \supseteq C^{hor}(I) \cap I^G$, which is equivalent to $C^{hor}_{ite}(I) \cap I^G \supseteq C^{hor}_{2s}(I) \cap I^G$.

Since the number of general-category individuals who get a position under C^{SCI} is weakly more than the number of general-category individuals who get a position under C^{hor}_{ite} and C^{hor} does not reject an individual unless the capacity is filled, the number of individuals with a reserve-eligible category who receive a position under C^{SCI} is weakly less than the number of individuals with a reserve-eligible category who receive a position under C^{hor}_{ite}. Similarly, the number of individuals with a reserve-eligible category who receive a position under C^{hor}_{ite} is weakly less than the number of individuals with a reserve-eligible category who receive a position under C^{hor}_{2s}.

Proof of Theorem 1. Part 1: When I is the set of applicants, let $I_1 \subseteq I$ be the set of individuals who are considered at Step 0 of C^{SCI} (equivalently, Step 0 of C^{hor}_{ite}), $I^o \subseteq I$ be the set of individuals who are allocated open-category positions by C^{SCI}, and $I^u \subseteq I$ be the set of individuals who are not allocated any positions by C^{SCI}.

To get $C^{SCI}(I) = C^{hor}_{ite}(I)$, we need to prove that C^{hor}_{ite} terminates at Step 3 when I is the set of applicants. Since Steps 0, 1, and 2 are the same in C^{SCI} and C^{hor}_{ite}, we need $C^{hor}(I^o \cup I^u | r^o, (r^o_t)_{t \in T}) = I^o$. We prove a more general result that $C^{hor}(I_1 \cup I^u | r^o, (r^o_t)_{t \in T}) = I^o$, which implies $C^{hor}(I^o \cup I^u | r^o, (r^o_t)_{t \in T}) = I^o$ because $I_1 \supseteq I^o$ and the fact that C^{hor} satisfies the irrelevance of rejected individuals. For the rest of the proof, we use C^{hor} with parameters $(r^o, (r^o_t)_{t \in T})$, and, to simplify the notation, we omit them.

For any individual $j \in I^u$ with a reserve-eligible category who does not have a trait, there are at least r^o number of individuals in I_1 who have higher merit scores than j. Therefore, individual j cannot be chosen by C^{hor} when $I_1 \cup I^u$ is the set of applicants.

For any individual $j \in I^u$ with a reserve-eligible category and a trait, say t, there are at least $\min\{r^o_t, |\{i \in I : t(i) = t\}|\}$ number of individuals with trait t in I^o who have strictly higher merit scores than j. This holds because $C^{SCI}(I)$ satisfies horizontal reservations for I and it eliminates justified envy. Furthermore, there are at least r^o individuals in I_1 who have higher merit scores than j. Therefore, an individual in I^u with a reserve-eligible category cannot be chosen when $I_1 \cup I^u$ is the set of applicants.

Since C^{hor} satisfies the irrelevance of rejected individuals, $C^{hor}(I_1 \cup I^u) = C^{hor}(I_1)$ because I_1 includes all general-category individuals and no individual in I_u with a reserve-eligible category can be chosen. By construction $C^{hor}(I_1) = I^o$, so $C^{hor}(I_1 \cup I^u) = I^o$. The conclusion follows that $C^{hor}_{ite}(I) = C^{SCI}(I)$.

Part 2: Suppose $C^{SCI}(I)$ does not eliminate justified envy and there is one trait t. Since there are sufficiently many individuals to fill the positions at each vertical category, $|C^{SCI}(I)| = q$. Then the instances of justified envy involve a trait-t individual with a reserve-eligible category who is rejected and a general-category individual with the same trait who is chosen.

Let I_t^G be the set of general-category individuals with trait t who are allocated an open-category position and I_t^R be the set of individuals with a reserve-eligible category and trait t who are unassigned such that every individual in I_t^G is justifiably envied by someone in I_t^R and every individual in I_t^R justifiably envies someone in I_t^G. Let k_t be the maximum integer such that the individual in I_t^G with the k-th lowest merit score is lower than the individual in I_t^R with the k-th highest merit score. Since $C^{SCI}(I)$ does not eliminate justified envy, $k_t > 0$. By construction, $|C^{SCI}(I) \setminus C^{hor}_{ite}(I)| = k_t$ because $C^{hor}_{ite}(I)$ replaces k_t individuals in I_t^G with the lowest merit scores, denote this set by A_t, with k_t individuals in I_t^R with the highest merit scores, denote this set by B_t.

Let C be a choice rule such that $C(I)$ eliminates justified envy.

ii(

We consider two cases. If $A_t \cap C(I) = \emptyset$, then $C^{SCI}(I) \setminus C(I) \supseteq A_t$. Therefore, $|C^{SCI}(I) \setminus C(I)| \geq |A_t| = k_t = |C^{SCI}(I) \setminus C^{hor}_{ite}(I)|$. Otherwise, if $A_t \cap C(I) \neq \emptyset$, then $C(I) \setminus C^{SCI}(I) \supseteq B_t$ because $C(I)$ eliminates justified envy and every individual in B_t has a reserve-eligible category, trait t and a higher merit score than all individuals in A_t who have general category and trait t. Since $|C^{SCI}(I)| = q$, $|C(I)| \leq q$, and $|C(I) \setminus C^{SCI}(I)| \geq |B_t| = k_t$, we get $|C^{SCI}(I) \setminus C(I)| \geq k_t = |C^{SCI}(I) \setminus C^{hor}_{ite}(I)|$.

. lr182 11.9552 Tf 13.761 765.045 0 Td [(i)]TJ/F1i)such that

Proof of Proposition 6. and Suppose, for contradiction, that C is incentive compatible, respects inter se merit, and satisfies the irrelevance of rejected individuals but it does not eliminate justified envy. Then, there exist $I \subset \mathcal{I}$, $i, j \in I$ with $r(i) \subseteq r(j)$, $t(i) \subseteq t(j)$, and $s(i) < s(j)$ such that $i \in C(I)$ and $j \in R(I)$. By incentive compatibility, if j withholds some of her reserve-eligible attributes and treated as an individual with category $r(i)$ and set of attibutes $t(i)$, then she will still not be chosen. lr182 11.9552 Tf 13.761 765.045 0 Td [(i)]TJ/F1i)

at the expense of a second displaced individual m_2. At this point, both horizontal reservation constraints are satisfied, and the outcome of the SCI-VHR choice rule is finalized as

$$\{m_1, w_1, m_4\}.$$

Observe that the same outcome is obtained if the disabled horizontal reservation is accommodated first and the female horizontal reservation is accommodated next.

Therefore, through the adjustment phase, two higher merit-score individuals m_2 and m_3 are removed from the original merit-based choice set. We argue that the removal of the individual m_2 is unjustified since both horizontal reservation constraints could have been accommodated with only one adjustment, namely by including the disabled female individual w_2 at the expense of the individual m_3. When the SCI-VHR choice rule was originally introduced, the judges of the Supreme Court in Anil Kumar Gupta (1995)indicated that, for the purpose of accommodating the horizontal reservations "the requisite number of special reservation individuals shall have to be taken and adjusted / accommodated against their respective social reservation categories by deleting the corresponding number of candidates therefrom." Since both of the special horizontal reservations can be satisfied with the inclusion of the disabled female individual w_2, we argue that the requisite number is only one. The outcome that has to be selected with only one adjustment is

$$\{m_1, m_2, w_2\}.$$

But this outcome cannot be achieved by accommodating the horizontal reservation types one at a time. Instead, a forward-looking approach is needed for the adjustment phase.

Appendix D. Case Study: Ashish Sharma & Ors. vs. State Of Chhattisgarh & Ors. on August 18th, 2003

In this Chhattisgarh High Court case, the petitioners challenge the implementation of horizontal reservations for women at a Chhattisgarh Medical School. There are 42 open seats, of which 13 are horizontally reserved for women, one is horizontally reserved for soldiers, and one is horizontally reserved for freedom fighters. In order to allocate the 42 open seats, the respondents followed a procedure that is mechanically different from the procedure for SCI-VHR choice rule C^{SCI}: They first allocated 13 seats to the highest merit score women, next allocated 27 seats to the remaining highest score candidates bringing the total to 40, and since horizontal reserves for soldiers and freedom fighters were not satisfied by this point, they assigned one seat each to the remaining candidates with the