Algebra Qualifying Examination, Fall 2019

Instuctions: This is a 3 hour examination. In the problems below, all rings are commutative with identity unless specified otherwise. This is a closed book exam, also no notes, searching the web, or otherwise consulting external sources. Good luck!

1. Let P be a finite p-group. Show that P is not cyclic if and only if P has a quotient isomorphic to $\mathbb{Z}/p\mathbb{Z} \times \mathbb{Z}/p\mathbb{Z}$.

2. Let R be a commutative ring with unity.

   (a) Let S be a non-empty saturated multiplicative set in R, i.e. if $a, b \in R$, then $ab \in S$ if and only if $a, b \in S$. Show that $R - S$ (the complement of S in R) is a union of prime ideals.

   (b) Suppposerythat R isuapdomain such that eAvenrythogzeroupGLN(C) 92.0 1 1(Tf33ite] 0-1(T33a,conju

   diagonal matrix.

   (b) If F is an algebraically closed field and $A \in GL_N(F)$ is of finite order, is