2 q^q 2.]

Exercise 2. Let R be an integral domain that is integrally closed in its eld of fraction F.

(1) Show that an algebraic is integral over R if and only if its minimal polynomial over F is a monic polynomial in $R[x]$.
(2) Show that for any monic $f(x)$ 2 $R[x]$, for any decomposition $f(x) = f_1(x)f_2(x)$ into monic polynomials in $F[x]$, the factors $f_1; f_2$ have coe -cients in R.

Exercise 3. Let k be an algebraically closed eld. Consider the a ne variety $V = k^2$ (with coordinates $x; y$), and the a ne variety $W = k^2$ (with coordinates $s; t$). Suppose ' : V ! W is a morphism, and denote by R $k[x; y]$ the image of the induced ring homomorphism ' : $k[s; t]$! $k[x; y]$. For each of the following statements, give a proof or a counterexample.

(1) If ' has Zariski dense image, then ' is surjective.
(2) If $k[x; y]=R$ is [x; y] an integral extensionJ/F11 ,4(in)25[(k)]gs-333(then)]TJ/F11 9.9626 Tf 132.nl1Ik

Exercise 7. Suppose k be a eld and $R = k[x; y; z]$ a polynomial ring. Compute

$$\mathrm{Ext}^i_R\big(R{=}(xz); R{=}(xy; xz)\big)$$

for all i 0.

Exercise 8. Suppose p is a prime of the form $4k + 3$. Find the conjugacy class of every element of order 4 in $\mathrm{GL}_2(\mathrm{F}_p)$.