



# Introduction

The origins of aggregate fluctuations are of essential interest to modern macroeconomics, as reaffirmed by the recent financial crisis and ensuing recession. A large literature has sought to explain the role of financial factors in the context of the financial accelerator mechanism, relying on representative agent assumptions in which a creditor lends to a borrower. This, however abstracts from the credit relationships amongst heterogeneous borrowers and lenders that characterizes most advanced economies. Yet the credit linkages between firms may propagate firm-level shocks across the economy. The literature has therefore overlooked a potentially important source of aggregate fluctuations, and is in need of a framework for evaluating whether the credit relationships between non-financial firms play a role in the business cycle.

To this end, I build a tractable model of a credit network economy in which trade in intermediate goods is financed by supplier credit. I show analytically how the trade credit linkages between non-financial firms generate aggregate fluctuations from firm-level shocks, and show that the mechanism is quantitatively important. I combine firm-level balance sheet data and industry-level input-output data to construct a proxy of supplier credit flows at the industry-level. I use this proxy to calibrate my model, and quantitatively analyze how the aggregate impact of idiosyncratic shocks depends on the structure of the credit network. I then use a structural factor approach to estimate the shocks which hit the US manufacturing and mining sectors over the period 1997-2013. Second, I use the model to shed light on the origins of aggregate fluctuations in the US by decomposing observed movements in industrial production (IP) into components arising from four types of shocks: aggregate productivity, idiosyncratic productivity, aggregate liquidity, and idiosyncratic liquidity shocks.

In so doing, I make two contributions to the literature. First, I show that the c(

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is the single most important source of short-term external finance for firms, accounting for more than half of firms' short-term liabilities and more than one-third of their total liabilities in most OECD countries.<sup>2</sup> In the US, accounts payable was three times as large as bank loans and fifteen times as large as commercial paper outstanding, on the aggregate balance sheet of non-financial corporations in 2012.<sup>3</sup> All of these facts point to the presence of strong credit linkages between non-financial firms.

An important feature of trade credit is that it leaves suppliers exposed to the liquidity problems of their customers. A notable example of this is the US automotive industry in 2008, when the Big Three automakers (Chrysler, Ford, and GM) faced a serious shortage of liquidity. While Ford did not require a bailout, it requested one from the US Congress on behalf of its competitors, fearing that a bankruptcy by Chrysler or GM would transfer the liquidity shortage to their common suppliers, as the money owed to them could not be paid until they exited bankruptcy. This episode suggests that when firms play a dual role of supplier and creditor, a shock may not only affect trade directly, but also the availability of liquidity to finance the trade.

There is growing evidence to suggest that this intuition is empirically relevant. A number of studies - including Boissay and Groen (2012), Jacobson and von Schedvin (2015), and Raddatz (2010) - have found that firm- and industry-level trade credit linkages propagate liquidity shocks from firms to their suppliers. In spite of this evidence, the macroeconomic implications of trade credit have been largely overlooked in the literature. I therefore develop a framework for understanding how inter-firm trade and credit interact in response to credit conditions.

I consider an economy in which firms are organized in a production network and trade intermediate goods with one another. Each intermediate good is produced using labor and other intermediate goods. There is

A firm-level liquidity shock propagates to other firms in the network via two channels. First is the standard input-output channel which has been the focus of studies such as Acemoglu et al. (2012) and Bigio and La'O (2013): the shocked firm cuts back on production, reducing the demand faced by its suppliers and reducing the supply of its good to its customers.

But the credit linkages between firms implies that there is a new channel of propagation - which I call the **credit linkage channel** - in which the shock directly affects the cash-in-advance payment received by the firm's suppliers. When the shocked firm cuts back on production, the price of its good rises, which increases the collateral value of its receivables. Able to obtain a higher trade credit loan (per unit of output) from its suppliers, the firm reduces the cash-in-advance payments it makes upstream. With less cash, the suppliers are more liquidity constrained, and they may themselves be forced to further cut back on their own production. If these suppliers cut back on production, they reduce their demand for labor, amplifying the aggregate effect

aggregate and idiosyncratic productivity shocks. A variance decomposition of aggregate IP shows that the credit network of these industries accounts for one-fifth of aggregate IP volatility.

Much of the previous literature has relied on aggregate productivity shocks to drive the business cycle. Yet by many accounts, this has been an unsatisfactory explanation due to the lack of direct evidence for shocks. This paper shows, however, that when one takes into account the credit linkages between non-financial firms in the economy, the role of aggregate productivity shocks is minimal. On the contrary, aggregate **liquidity** shocks seem to play a vital role the business cycle. Indeed, the importance of shocks emanating from the financial sector to real economy as a whole is well-documented. Thus, this paper suggests that a large fraction of aggregate fluctuations are perhaps driven by shocks from the financial sector emanating to the real economy.

The rest of the paper is organized as follows. The next section reviews some of the literature to which this paper is related. Part I introduces the model. The first part of the model considers a simple version in which the structure of the production network is a supply chain. I derive analytical results using a stylized version of the full model. In the next part, I generalize the production network structure. Part II is a quantitative analysis. I describe the proxy of trade credit flow, the calibration, and quantitative results. In Part III, I perform my empirical analysis, and discuss the results.

## Literature Review

(In progress).

This paper relates to several strands of the literature. There is a large literature on the role of financial frictions in macroeconomics. Studies such as Bernanke and Gertler (1995), Bernanke et al. (1999), and Kiyotaki and Moore (1997b) evaluate the link between financial factors and the real economy. Most of this literature abstracts from heterogeneous agents models. Also, there has been little attention given to the credit relationships between non-financial firms. I consider a financial accelerator mechanism in the context of a network model and show that amplifies its effects.

A growing literature looks to network effects as a multiplier mechanism which can generate aggregate fluctuations from idiosyncratic shocks. Much of this literature builds on the multi-sector RBC model of Long and Plosser (1983). Most notably, these include Acemoglu et al. (2012), Shea (2002), Duor (1999), Horvath (1998), Horvath (2000), and Acemoglu et al. (2015). These studies all focus on the role of input-output linkages between firms. Input-specificity in the production of intermediate goods prevents firms from easily switching suppliers or customers in response to productivity shocks. Generally, these models rely on certain structural properties of a network in which idiosyncratic shocks to firms in economy do not average out. Systemically important firms, who take a central role in the network, propagate shocks across other firms in the network generating movements at the aggregate level of the economy. However, most of this literature do not model how trade in intermediate goods is financed. Indeed, most abstract away from financial frictions.

A notable work to which this paper is most closely related is that of Bigio and La'O (2013), who examine



## Part I

# Model

In Part I, I introduce and analyze the model. This section has two parts. For ease of exposition, it is instructive to first consider the special case of a vertical production network. I refer to this as the **stylized model**. The analytical tractability of this case permits closed-form expressions for aggregate output. In the second part, I generalize the network structure.

## 1 Stylized Model: Vertical Production Structure

### 1.1 Economic Environment

There is one time period, consisting of two parts. At the beginning of the period, contracts are signed. At the end of the period, production takes place and contracts are settled. There are three types of agents: a representative household, firms, and a bank. There are  $M$  goods, each produced by a different firm. (Here the productive unit could similarly be called an industry, which is composed of a continuum of competitive firms). Each good can be consumed by the household or the production of other goods.

### 1.2 Representative Household

goods, household preferences

$$C = wN + \sum_{i=1}^M p_i x_i \quad (1)$$

### 1.2.2 Optimality

The household's optimality condition is given by

$$\frac{V^q(N)}{U^q(C)} = w \quad (2)$$

This equates the competitive wage with the marginal rate of substitution between labor and consumption.

### 1.3 Firms

There are  $M$  firms who each produce a different good. Suppose for now that firms are arranged in a supply chain, where each firm produces an intermediate good for one other firm. The last firm in the chain produces the consumption good, which it sells to the household. Firms are indexed by their order in the supply chain, with  $i = M$  denoting the producer of the final good.

Firms are price-takers.<sup>4</sup> The production technology of firm  $i$  is Cobb-Douglas over labor and intermediate goods.

$$x_i = \begin{cases} z_i n_i^{\alpha_i} & \text{for } i = 1 \\ z_i n_i^{\alpha_i} x_{i-1}^{(1-\alpha_i)\beta_{i-1}} & \text{for } i > 1 \end{cases}$$

Here,  $x_i$  denotes firm  $i$ 's output,  $n_i$  its labor use, and  $x_{i-1}$  its use of good  $i-1$ . Parameter  $z_i$  denotes firm  $i$ 's total factor productivity,  $\alpha_i$  the share of labor in its production, and  $\beta_{i-1}$  the use of good  $i-1$  in firm  $i$ 's production. Let  $p_s$  denote the price of good  $s$ . The value of the sales from firm  $s$  to firm  $c$  is then  $p_s x_{cs}$ .

The input-output structure of the economy can be summarized by a matrix of input



$  \begin{matrix}  2 \\  \text{! } 11 & \text{! } 12 & \text{! } 13 \\  \text{! } 21 & \text{! } 22 & \text{! } 23 \\  \text{! } 31 & \text{! } 32 & \text{! } 33 \\  \vdots & & \\  \text{! } M1 & &  \end{matrix}  $	$  \begin{matrix}  3 & 2 \\  \text{! } 1M & 0 & 0 & 0 \\  \text{! } 21 & 0 & 0 & \\  \text{! } 32 & 0 & & \\  \vdots & & & \\  \text{! } MM & 0 & &  \end{matrix}  $	$  \begin{matrix}  3 \\  0 \\  \text{! } 21 \\  \text{! } 32 \\  \vdots \\  \text{! } M;M & 1 & 0  \end{matrix}  $
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**Bank lending:** Each firm chooses how much to borrow from the bank, subject to a limited enforcement problem. Firm  $i$  can obtain the loan  $b_i$  from the bank at the beginning of the period by pledging a fraction  $B_i$  of its total end-of-the-period revenue  $p_i x_i$ , and a fraction  $1 - B_i$  of its accounts receivable  $a_{i+1}$ , where  $B_i \leq 1$ . Thus, firm  $i$  faces a bank borrowing constraint of the form

$$b_i \leq B_i p_i x_i + (1 - B_i) a_{i+1}$$

Parameters  $B_i$  and  $a_{i+1}$  provide an exogenous source of liquidity to each firm, and represent the severity of the agency problem between firm  $i$  and the bank. I will later show that  $B_i$  parameterizes the degree of substitutability between bank credit and cash-in-advance payments from customers. Since  $b_i$  is chosen by firm  $i$  these bank borrowing constraint will bind in equilibrium as each firm obtains the maximum bank loan possible.

**Trade credit:** Each firm  $i$  chooses the size of the trade credit loan  $l_{i-1}$  it obtains from its supplier. But a limited enforcement problem between firms places a limit on the size of this loan. In particular, firm  $i$  can pledge a fraction  $\alpha_i$  of its end-of-the-period output to repay its supplier. Then the trade credit loan is bounded by the collateral value of firm  $i$ 's output

$$l_{i-1} \leq \alpha_i p_i x_i$$

The precise limited enforcement problem which reduces this borrowing constraint is described in detail in the Appendix. In equilibrium, the firm takes the maximum loan that the supplier will allow, and so the borrowing constraint binds. This pins down the trade credit loan from supplier  $i-1$  at  $l_{i-1} = \alpha_i p_i x_i$ . Note that the size of the loan to firm  $i$  depends on the price  $p_i$  of its good. (Hence, changes in the collateral value of good  $x_i$  will change the amount of cash-in-advance that supplier  $i-1$  can collect.)

The structure of the credit network between firms can be summarized by the matrix of  $\alpha_{ij}$ 's.

2				3
11	12	13	1M	
21	22	23	Z	
31	32	33	g.d [(ca494 Tdw0.38 495 Td22)]J/F7 6.178 T96.9738 Tf 2.81J/F15 9.	
⋮			⋱	
M 1			MM	

where

$$\lambda_i = \frac{b_i}{p_i x_i} + \frac{i_{i+1}}{p_i x_i} + 1 - \frac{i}{p_i x_i}$$

The variable  $\lambda_i$  denotes the **tightness** of firm  $i$ 's liquidity constraint. Notice that  $\lambda_i$  is decreasing in  $\frac{i}{p_i x_i}$ , the amount of  $i$ 's output sold on credit: the more credit that  $i$  gives its customer, the less cash it collects at the beginning of the period. We can replace  $\lambda_i$  using  $i+1$ 's binding supplier borrowing constraint, to re-write  $\lambda_i$ .

$$\lambda_i = B_i + \lambda_{i+1} \frac{p_{i+1} x_{i+1}}{p_i x_i} \quad (4)$$

Equation (4) shows that  $\lambda_i$  is an **equilibrium object**; it is an endogenous variable which depends on the revenue of firm  $i$  and firm  $i+1$ . Hence, changes in the price of its **customer's** good affect the tightness of firm  $i$ 's liquidity constraint. Note also that the dependence of  $\lambda_i$  on prices  $p_i$  and  $p_{i+1}$  means that changes in a shock will have **general equilibrium effects** on each  $\lambda_i$ .

inputs, it does not distort the firm's optimal choice of expenditure on labor versus the intermediate good. However, the constraint will limit the firm's **total** expenditure on both inputs.

If firm  $i$ 's liquidity constraint is not binding in equilibrium, then it simply maximizes its profit function. Its optimal level of expenditure on each input is determined by a condition which equates the marginal cost of the input with its marginal revenue product. The firm's expenditure on labor is therefore given by

$$w n_i = p_i x_i ; \quad p_{i-1} = \frac{1}{r_i} (1 - \alpha_i) \frac{p_i x_i}{x_{i-1}}$$

If, on the other hand, the constraint is binding in equilibrium, then the amount of liquidity  $p_i x_i$  that firm  $i$  has limits how much the firm can spend on **both** inputs. In particular, firm  $i$ 's expenditure on labor and good  $i-1$  is given by

$$w n_i = \frac{1}{r_i} p_i x_i ; \quad p_{i-1} = \frac{1}{r_i} (1 - \alpha_i) \frac{p_i x_i}{x_{i-1}}$$

I show in the Appendix that firm  $i$

Re-arranging this and replacing  $\frac{p_{i+1} x_{i+1}}{p_i x_i}$  in ( ) yields  $i$

$$Y = \prod_{i=1}^n P_i^{\gamma_i}$$

Thus, equilibrium aggregate output is log-linear in each firm's labor wedge, and equals  $Y$  if and only if  $\lambda_i = 1$  for all  $i$  - i.e. if no firm's liquidity constraint is binding in equilibrium.<sup>5</sup> captures the **aggregate liquidity** available to all firms in the economy for trade in inputs. Therefore, (7) says that equilibrium aggregate output is constrained by the aggregate liquidity in the economy at the beginning of the period. Notice that through  $\gamma$ , firms who are further downstream have a higher share of total employment through the use of intermediate goods, and therefore have a higher impact on aggregate liquidity.

### 1.4.1 Equilibrium Characterization

To summarize the equilibrium, the cash-in-advance constraints faced by firms induces a wedge on their production, which depends on the tightness of their constraints. But in a setting where firms share liquidity via trade credit, these wedges depend endogenously on the prices of downstream goods and the structure of the credit network. In the next section, I explore the implications of this endogenous relationship between wedges and prices for how aggregate output responds to firm-level shocks.

At this stage, it is worth discussing how this economy compares to that of Bigio and La'O (2013). The novelty of Bigio and La'O (2013) is to show how wedges aggregate in an input-output network. However, in Bigio and La'O (2013), all payments between firms are settled at the end of the period after production takes place. As a result, there is no role for trade credit; and  $\lambda_i$  and  $\mu_i$  are fixed exogenously. As I show in the next section, the endogeneity of the wedges means that the economy behaves qualitatively very differently in response to local shocks.

## 1.5 Aggregate Impact of Firm-Level Shocks

In this section, I examine the response of aggregate output to firm-level liquidity and productivity shocks.

### 1.5.1 Liquidity Shocks

I model a **liquidity shock** to firm  $i$  by a change in  $B_i$ , the fraction of firm  $i$ 's revenue that the bank will accept as collateral for the bank loan. Consider a marginal fall in  $B_i$  given by  $d B_i$ . This is a reduced-form way to capture an adverse shock to firm  $i$ 's bank which affects the ability of firm  $i$  to obtain credit for purchasing inputs.<sup>6</sup>

<sup>5</sup>Note that although  $Y$  is log-linear in each  $\lambda_i$ , it is not globally log-linear in  $\lambda_i$ . (This is reflected in the kink in  $\lambda_i$  at  $\lambda_i = r_i$ .) Why is  $Y$  not globally log-linear in  $\lambda_i$ ? The liquidity constraint creates a kink in the policy function for employment  $n_i$  at the point at which the liquidity constraint is no longer binding, i.e. at  $\lambda_i = r_i$ . This kink carries over to  $Y$  in aggregation. The kink implies: i)  $Y$  is not differentiable with respect to  $\lambda_i$  at  $\lambda_i = r_i$ .

globally log-linear

The fall in  $B_i$  directly affects the amount of cash firm  $i$  can raise at the beginning of the period. The closed-form expression for  $\lambda_i$  (4) shows that the fall in  $B_i$  causes firm  $i$ 's liquidity constraint to tighten.

$$\frac{d\lambda_i}{dB_i} = -1 < 0$$

If firm  $i$ 's liquidity constraint is binding in equilibrium, then the tighter liquidity constraint forces the firm







+

2) #  $i_1$  => drop in demand for all  $j < i_1$ ; drop in supply for all  $j > i_1$  =>  $Y$  falls

dli

+

3) #  $i_2$  => drop in demand for all  $j < i_2$ ; drop in supply for all  $j > i_2$  =>  $Y$  falls

⋮

In this manner, the credit linkages between firms trigger the standard in-out-out channel at every level of production, increasing the total demand/supply effects faced by each firm. Thus, a firm-level liquidity shock to in my model is isomorphic to an **aggregate** liquidity shock to all firms in a model with fixed wedges, e.g. Bigio and La'O (2013). I explore this point in further detail in the quantitative analysis.

### 1.5.2 Impact of Firm-Level Shock on Aggregate Output

I now formalize the network effects of the shock on aggregate output. Recall from (7) that equilibrium aggregate output is log-linear in each firm's wedge

$$\log Y = \log Y + \log$$

Then the elasticity of aggregate output with respect to firm  $i$ 's bank borrowing  $B_i$  is given by

$$d \log Y$$



### 1.5.3 Productivity Shocks

Now consider a productivity shock to firm  $i$ , represented by a fall in  $i$ 's total factor productivity (TFP)  $z_i$ . What is the effect on aggregate output? Recall the closed-form expression (7) for aggregate output

$$Y = \bar{Y}$$

where

$$\bar{Y} = \left( \sum_{j=1}^M \bar{\gamma}_j z_j^{\frac{1}{\sigma}} \right)^{\sigma} \quad \bar{Y} = \left( \sum_{j=1}^M \bar{p}_j \right)^{\frac{1}{1-\sigma}}$$

I claim that  $\bar{Y}$  is independent of  $z_i$ . To see this, first recall that  $\bar{p}_j = \min \{1, \frac{M}{\Gamma_M} g_j\}$ , where  $\bar{p}_j = \bar{p}_j$



$$\frac{p_i c_i}{p_j c_j} = \frac{i}{j}; \quad N^{1+} = C$$

## 2.2 Firm Liquidity

Each firm's liquidity constraint takes the same form as in the stylized model, with the exception that each firm has  $M$  suppliers and  $M$  customers instead of just one of each. Firm  $i$  is required to pay its wage bill  $wn_i$  and its intermediate good purchases  $p_s x_{is}$  from each supplier  $s$  in advance. It receives a loan  $b_i$  from the bank and a trade credit loan  $z_{is}$  from each supplier.

$$wn_i + \sum_{s=1}^M (p_s x_{is} - z_{is}) = b_i + \sum_{c=1}^M p_i x_{ic}$$

net CIA payment to suppliers
net CIA payment from suppliers

the quantitative predictions of the model, which I discuss later on.

Each firm chooses the size of the loan to obtain from each creditor, so that the borrowing constraints bind in equilibrium. Plugging the binding borrowing constraints into firm  $i$ 's liquidity constraint yields a constraint on  $i$ 's total input purchases

$$wn_i + \sum_{s=1}^{\mathcal{X}^M} p_s x_{is} = \lambda_i p_i x_i$$

where  $\lambda_i$  denotes the tightness of  $i$ 's liquidity constraint.

$$\lambda_i = B_i + \sum_{s=1}^{\mathcal{X}^M} \lambda_{is} + 1 = \sum_{c=1}^{\mathcal{X}^M} \lambda_{ci} \frac{p_c x_c}{p_i x_i}$$

Note that  $\lambda_i$  is again an equilibrium object, depending on the prices customers' goods  $p_c$  and forward credit linkages  $\lambda_{ci}$  for all  $c$ .

TABLE SUMMARIZING DEFINITIONS OF PARAMETERS AND EQ. VARIABLES

## 2.3 Firm Optimality Conditions and Market Clearing

Firms choose labor and intermediate goods to maximize profits subject to their liquidity constraint. This yields optimality conditions of the same form, equating the ratio of expenditure on each good with the ratio of their marginal revenue products.

$$\frac{wn_i}{p_j x_{ij}} = \frac{\lambda_i}{(1 - \lambda_i) \lambda_{ij}}$$

Again, the liquidity constraint of firm  $i$  inserts a wedge  $\lambda_i$  between the marginal cost and marginal revenue product of each input

$$n_i = \lambda_i \frac{p_i}{w} x_i \quad x_{ij} = \lambda_i (1 - \lambda_i) \lambda_{ij} \frac{p_i}{p_j} x_i \quad (10)$$

where the wedge depends on the tightness of  $i$ 's constraint and its returns-to-scale.

$$\lambda_i = \min \left\{ 1; \frac{1}{r_i} \right\}; \quad r_i = \lambda_i + (1 - \lambda_i) \sum_{j=1}^{\mathcal{X}^M} \lambda_{ij} \quad (11)$$

Note that the wedge is still an equilibrium object, depending on collateral value of each customer's output and forward credit linkages. Endogenous wedges in any equilibrium will take same form, and will respond in qualitatively same way as previously.

Market clearing conditions for labor and each intermediate good are given by

$$N = \sum_{i=1}^M n_i \quad x_i = q_i + \sum_{c=1}^M x_{ci}$$





### 3.1 Data

To build my proxy, I use two sources of data: input-output tables from the Bureau of Economic Analysis (BEA) and Comustat North America over the sample period 1997-2013. The BEA publishes annual data on commodity use by industry (Uses by Commodity Table) at the three-digit level of the North American Industry Classification System (NAICS). At this level, there are 58 industries, excluding the financial sector. From this data, I observe annual trade flows between each industry-pair, which corresponds to  $p_j x_{ij}$  in my model for every industry-pair  $f; j \in \mathcal{G}$ . The BEA also publishes an annual Direct Requirements tables at the same level of detail, which indicate for each industry the amount of a commodity that is required to produce one dollar of that industry's output. These values are quite stable over my sample period. In constructing my proxy, and also in calibrating the model later, I use the input-output tables of the median year in my sample, 2005.

Comustat collects balance-sheet information annually from all publicly-listed firms in the US. The available data includes each firm's total accounts payable, accounts receivable, cost of goods sold, and sales in each year of the sample. Therefore, while I cannot identify from whom each firm receives trade credit or to

$$\text{PayFin}_{f,t} = \frac{.5(\text{AP}_{f,t-1} + \text{AP}_{f,t})}{\text{COGS}_{f,t}}$$

I do this only for years in which there is data for both AP and COGS for each firm. I obtain a firm-level measure of payables financing by taking the median of  $\text{PayFin}_{f,t}$  across time, to minimize effect of outliers and get a representative firm-level estimate of the average COGS financed with trade credit. Then to get an industry-level measure of payables financing, I take the median of  $\text{PayFin}_f$  across all firms  $f$  in each three-digit level NAICS industry. In this way, I obtain a measure of payables financing for each of my industries.

Raddatz (2010) uses this industry-level measure of PayFin to construct  $\mathbf{q}_j$ . However, since he only uses AP data, he must impose that  $\mathbf{q}_j = \mathbf{q}_k$  for all  $j, k$ . In other words, he assumes that each industry finances the same fraction of purchases with trade credit, across all of its suppliers. This is a fairly strong assumption

### 3.3 Choosing a Proxy

In this section, I consider an alternative weighting scheme for building the proxy  $\mathbf{q}_j$  and compare it with my baseline weighting scheme. Let  $F_B(\text{PayFin}_i; \text{RecLend}_j)$  denote the baseline weighting function for building  $\mathbf{q}_j$ , in which weights are assigned each argument according to the size of each industry.

$$F_B(\text{PayFin}_i; \text{RecLend}_j) = \frac{p_i x_i}{p_i x_i + p_j x_j} \text{PayFin}_i + \frac{p_j x_j}{p_i x_i + p_j x_j} \text{RecLend}_j$$

The alternative I consider is  $F_A$ , in which I assign equal weights to the arguments.

$$F_A(\text{PayFin}_i; \text{RecLend}_j) = \frac{1}{2} \text{PayFin}_i + \frac{1}{2} \text{RecLend}_j$$

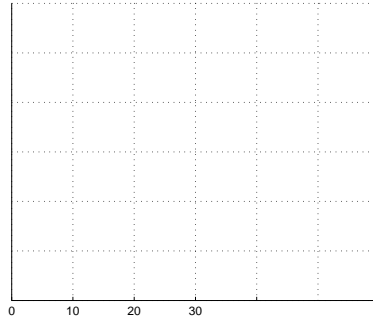
$F_B$  and  $F_A$  are equivalent when all industries have the same revenue. To the extent that there is greater variation in the size of industries, the two weighting schemes will produce different proxies for  $\mathbf{q}_j$ . Since the variation in the observed size distribution of industries is non-negligible, I need a metric by which to choose between  $F_B$  and  $F_A$ .

Recall that the measures  $\text{PayFin}_i$  and

$j\text{RecLend}_i = F_P(\text{PayFin}_c; \text{RecLend}_i)$



Figure 1:



These two measures respectively measure how much trade credit an industry provides the rest of the economy,

Figure 2:

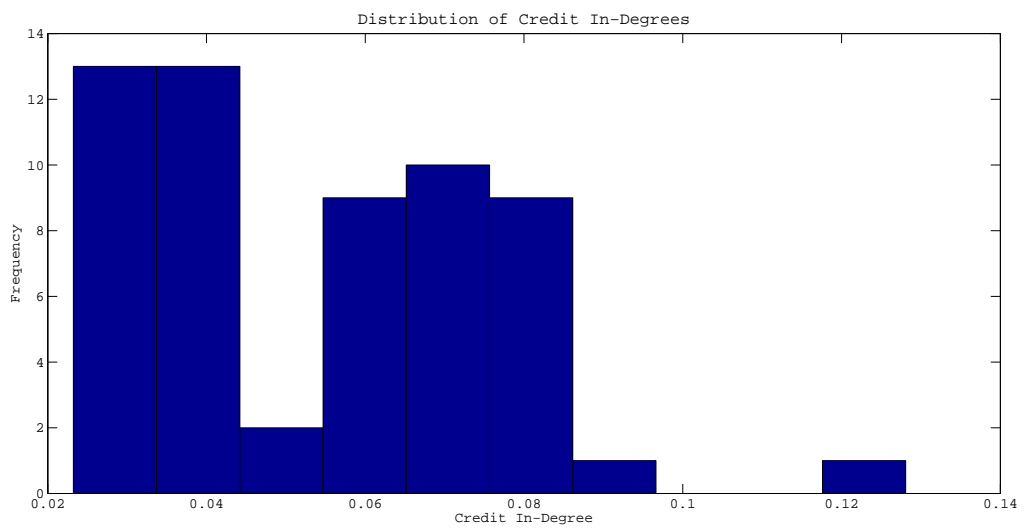
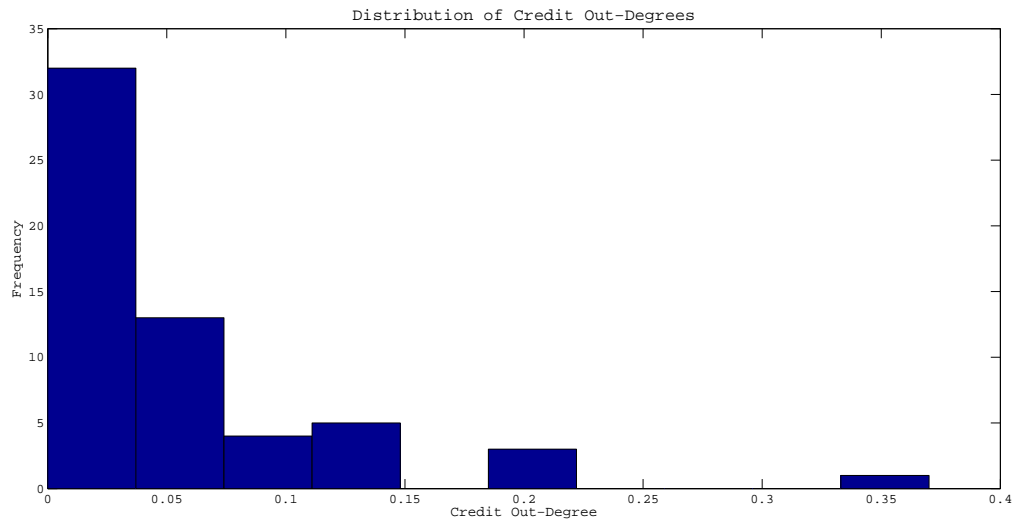
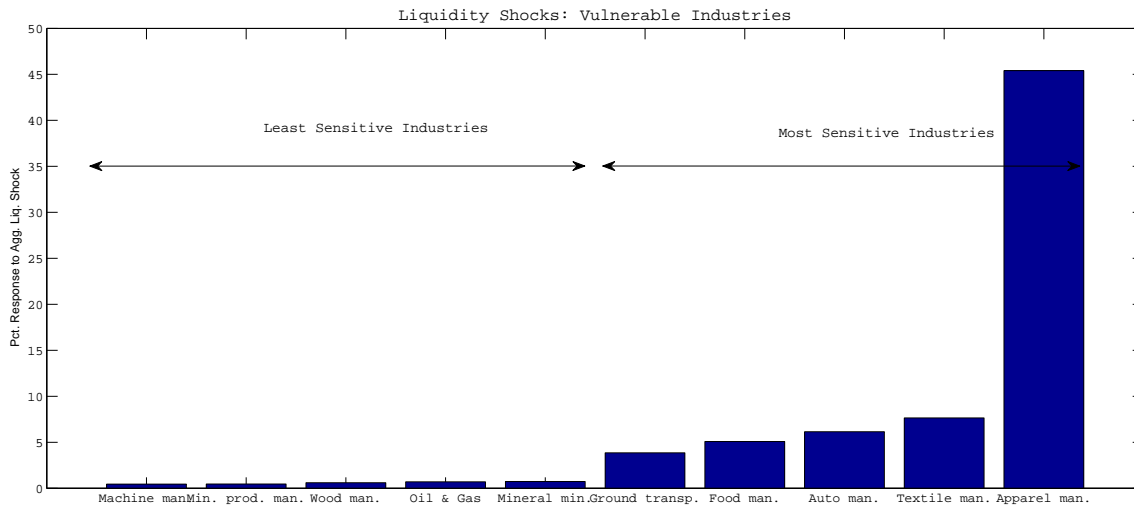






Figure 3:



	Total	With credit linkage channel Shut-Off
Pct. Fall in Y	3.15	2.1

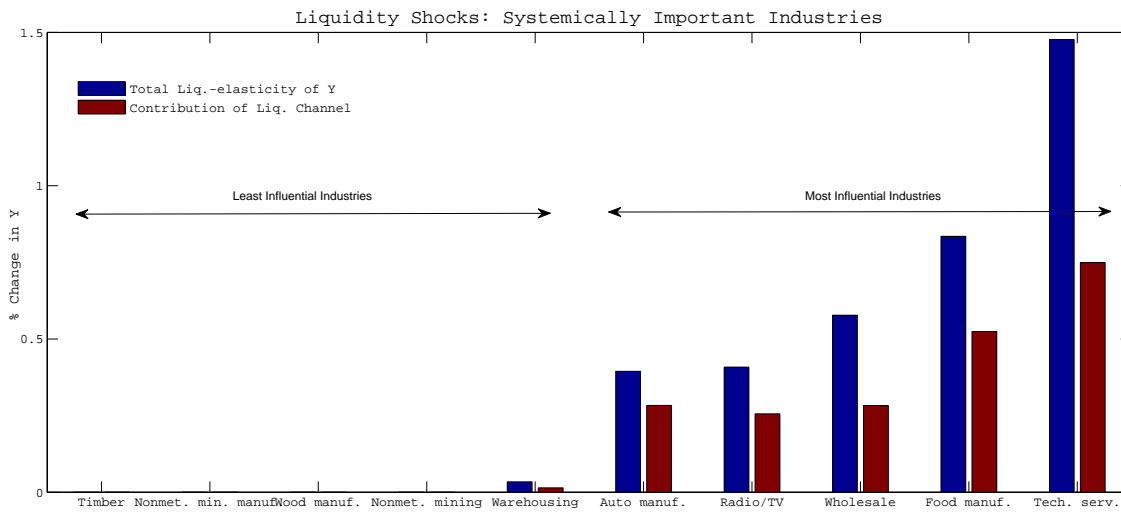
is one of the most vulnerable industries. (Will expand on this).

### 5.1.2 Results for $\alpha = .8$

Next, I perform the same exercise for with  $\alpha = .8$ , allowing for substitutability between bank credit and cash-in-advance payments. Table 2 reports the results.

Even in this more conservative case, the aggregate impact of the shock is quite large: Y falls by 3.15 percent. Although the amplification generated by the credit network falls substantially, it is still quantitatively relevant. The credit linkages between industries reduce a larger drop in Y by .54 percentage points. Put differently, the credit network of the US accounts for 17.1 percent of the drop in GDP in response to the aggregate liquidity shock. Therefore, even allowing for firms to substitute lost payments with increased bank borrowing does not substantially diminish the effect of credit linkages in generating aggregate fluctuations. The remainder of the paper uses  $\alpha = .8$ .

Figure 4:



## 5.2 Industry-Level Liquidity Shocks

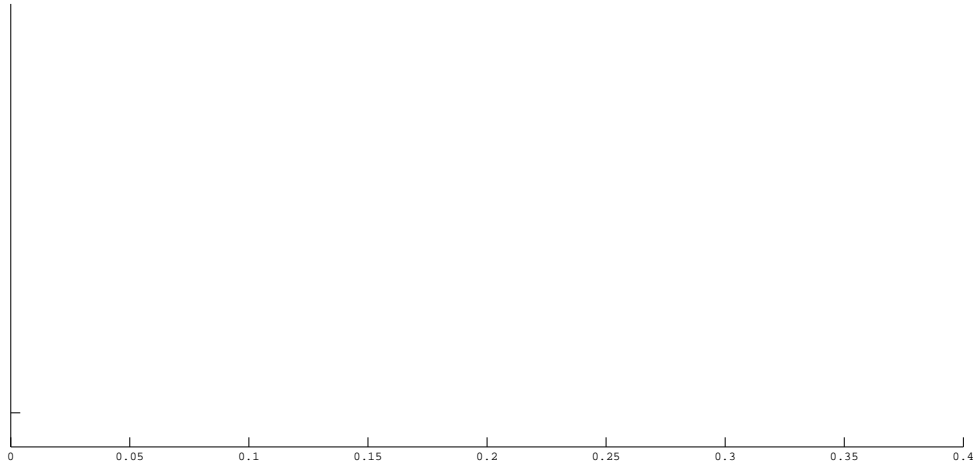
Next, I ask which industries are most systemically important in the economy, and how this relates to their position in the credit network. I measure the systemic importance of industry  $i$  by the elasticity of aggregate output with respect to its liquidity  $B_i$ .<sup>9</sup> A higher elasticity implies that an industry-level liquidity shock to  $i$  has a larger impact on aggregate output.

Figure 4 shows a bar graph of the five most and five least systemically important industries in the US. The blue bars show the elasticity of aggregate output with respect to each industry's liquidity, or the percentage drop in  $Y$  following a 1 percent drop in  $B_i$ .

The red bars show the contribution of the full credit network to each elasticity, which is computed by subtracting the drop in  $Y$  that occurs with credit linkage channel shut off, from the total drop in  $Y$ . To shut off the credit linkage channel, I impose that each industry's wedge  $\phi_i$  changes only in response to a direct liquidity shock to that industry, and not endogenously through credit linkages with other industries.

.09 percent of US GDP, a one percent liquidity shock this industry causes a fall in GDP of .19 percent, due to its input-output and credit linkages with other industries. This is an enormous response in aggregate output. In the absence of any linkages, the elasticity of GDP to this industry's liquidity would be equal to its share of GDP; i.e. GDP would fall by only .09 percent in response to this shock. Therefore, the

Figure 5:



industries. On average, increasing the credit out-degree of  $i$ 's most important supplier increases  $i$ 's systemic influence by \_\_\_ percentage points. This is a quantitatively significant effect, indicating that the aggregate impact of an industry-level shock to industry  $i$  depends strongly on how much liquidity  $i$ 's main suppliers provide to the rest of the economy.

The reason for this was elucidated by the analytical results of the stylized model, and can be understood in two steps. Suppose industry  $i$  experiences a liquidity shock to  $\mathbf{B}_i$ , and suppose that its most important supplier is industry  $j$ . First, the liquidity shock to  $i$  acts as a supply shock to each of its  $M$  customers, which increases the price of these customers' goods. This increase in price increases the collateral value of each customer's output. Second, since industry  $j$  also supplies goods to these  $M$  industries, the increase in collateral value means that  $j$  collects cash-in-advance becomes more constrained. Industry  $j$  then passes this shock to the rest of the economy, and so on. The stronger that industry  $j$ 's downstream credit linkages are with other industries, i.e. the higher its credit out-degree, the stronger this effect is, and the greater the aggregate impact of the shock to  $\mathbf{B}_i$ . The mechanics of this is explained in detail in the Appendix using the log-linearized equations.

## 5.4 Summary of Quantitative Analysis

The quantitative analysis showed that i) the credit linkages between US industries play a quantitatively

how important for the economy its suppliers are in providing credit.

Therefore an understanding of the role that credit linkages play in propagating idiosyncratic shocks introduces a new notion of the systemic importance of firms or industries based on their place in the credit network. The effects of these linkages are quantitatively important. Therefore, by overlooking the importance of credit linkages between nonfinancial firms, the literature has missed an important determinant of what makes an industry or firm systemically important.

## 5.5 Aggregate Productivity Shock

### Part III

# Empirical Analysis

Now that I have established the role that the credit network plays in propagating shocks, and shown that it can play a quantitatively significant role in generating fluctuations in aggregate output by amplifying

## 6.1 Data

From the Federal Reserve Board's Industrial Production Indexes, I observe the growth rate in output of all

affect wedges, but directly affect the the amount of labor employed **per unit** of output produced. The model uses these differential effects to identify the source of fluctuations in observed output and employment.

To see this, recall the production functions, optimality conditions for labor use, and definition of the wedges. First, the employment and output of an industry are linked by the industry production function  $x_{it} = z_{it} n_{it}^i \prod_{s=1}^M x_{ist}^{1-\alpha_i}$ . Therefore, a change in the TFP of industry  $i$  is given by

$$z_{it} = x_{it} / (n_{it}^i \prod_{s=1}^M x_{ist}^{1-\alpha_i})$$

The constant returns-to-scale of industry  $i$ 's production function implies that if an observed change in industry  $i$ 's output  $x_{it}$  from period  $t-1$  to  $t$  exceeds that of  $n_{it}^i \prod_{s=1}^M x_{ist}^{1-\alpha_i}$ , then there must have been an increase in  $i$ 's TFP such that  $z_{it} > 0$ .

Industry  $i$ 's optimality condition for labor equates the ratio of its marginal bill to revenue with labor's marginal product, times the wedge, i.e.  $\frac{w_{it}}{p_i x_{it}}$



### 6.3 Using the Model to Back Out Shocks from the Data

Recall that equations (1)-(2) are a system of log-linear equations describing the (first-order approximated) elasticity of each equilibrium variable to the liquidity  $\mathbf{B}_i$  and productivity  $\mathbf{z}_i$  of each industry  $i$ . Suppose that the static model is extended to be a repeated cross-section. Then equations (1)-(2) describe the evolution of the equilibrium variables that occurs each period in response to liquidity and productivity shocks, to a first-order approximation. I obtain a closed-form solution for this evolution, which is derived in the Appendix.

Let  $\mathbf{X}_t$  and  $\mathbf{N}_t$  denote the  $M$ -by-1 dimensional vectors of industry output and employment growth at time  $t$ ,  $\mathbf{x}_{it}$  and  $\mathbf{n}_{it}$ , respectively. And let  $\mathbf{B}_t$  and  $\mathbf{z}_t$  similarly denote the  $M$ -by-1 dimensional vectors of industry liquidity and productivity growth (i.e. shocks) at time  $t$ ,  $\mathbf{b}_{it}$  and  $\mathbf{z}_{it}$ , respectively. The closed-form solutions for  $\mathbf{X}_t$  and  $\mathbf{N}_t$  yield

$$\mathbf{X}_t = \mathbf{G}_X \mathbf{B}_t + \mathbf{H}_X \mathbf{z}_t$$

$$\mathbf{N}_t = \mathbf{G}_N \mathbf{B}_t + \mathbf{H}_N \mathbf{z}_t$$

These respectively describe how each industry's output and employment changes each period in response to the liquidity and productivity shocks to every industry. Here, the  $M$ -by- $M$  matrices  $\mathbf{G}_X$ ;  $\mathbf{G}_N$ ;  $\mathbf{H}_X$  and  $\mathbf{H}_N$  are functions of the economy's input-output and credit networks  $\mathbf{A}$  and  $\mathbf{C}$ , and capture the effects of the input-output and credit linkages in propagating either type of shock across industries, as was described in the theoretical analysis. The elements of these matrices depend only on the model parameters, and therefore take their values from my calibration.

I construct  $\mathbf{X}_t$  and  $\mathbf{N}_t$  for US industrial production industries (at the three-digit NAICS level) from the output and employment data described above. Let  $\hat{\mathbf{X}}_t$  and  $\hat{\mathbf{N}}_t$  denote these observed fluctuations. I then have a system of  $2M$  equations in as many unknowns for each quarter, and can invert the system to back-out shocks  $\mathbf{B}_t$  and  $\mathbf{z}_t$  each quarter from 1997 Q1 to 2013 Q4.

$$\mathbf{B}_t = \mathbf{G}_N^{-1} \hat{\mathbf{N}}_t - \mathbf{H}_N \mathbf{z}_t$$

$$\mathbf{z}_t = \mathbf{Q}^{-1} \hat{\mathbf{X}}_t - \mathbf{Q}^{-1} \mathbf{G}_X \mathbf{G}_N^{-1} \hat{\mathbf{N}}_t$$

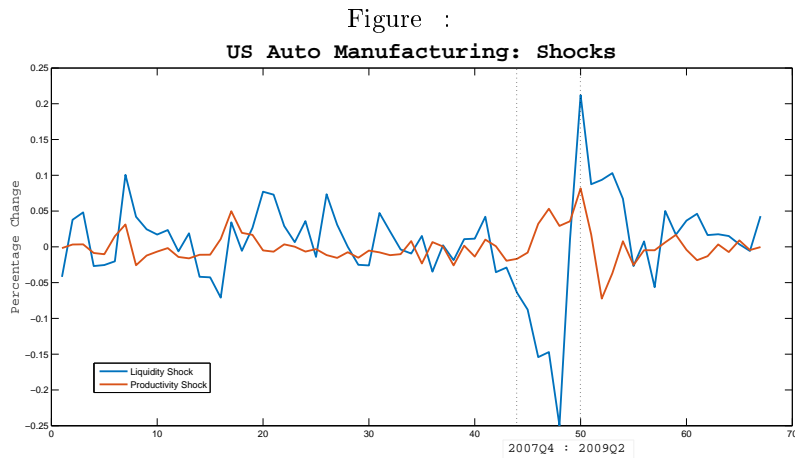


Figure shows the time series of the estimated liquidity and productivity shocks which hit the US auto manufacturing industry each quarter over the sample period.

From the figure, we can see that the changes in auto manufacturing’s liquidity and productivity both fluctuate moderately around zero for most of the sample period. Between 2007 and 2009, the liquidity available to this industry took a sharp drop for a number of consecutive quarters, reaching up to a 25 percent decline. Over this period, the industry’s output and employment experienced a large drop attributable to changes in the labor wedge of the industry. Given the credit linkages, the model is able to trace how much of the drop in the wedge is due to a direct liquidity shock to auto manufacturing versus shocks to other industries being transmitted to it. The blue line plotted in the figure reflect the direct liquidity shocks experienced each quarter by the industry.

In addition, the TFP of the industry seems to have not fluctuated greatly over this recessionary period; in fact, it increased slightly. These features broadly hold across most industries in industrial production. The aggregate effects of these features and their interaction will be discussed in subsequent sections.

## 6.4 Dynamic Factor Analysis

Next, I decompose the changes in industry liquidity and productivity,  $\mathbf{B}_t$  and  $\mathbf{z}_t$ , into an aggregate and industry-level shock. I assume that each may be described by a common component and a residual idiosyncratic component.

$$\mathbf{B}_t = \mathbf{B} \mathbf{F}_t^{\mathbf{B}} + \mathbf{u}_t$$

$$\mathbf{z}_t = \mathbf{z} F_t^Z + \mathbf{v}_t$$

Here,  $F_t^B$  and  $F_t^Z$  are scalars denoting the common factors affecting the output and employment growth of each industry, respectively, at quarter  $t$ . I interpret these factors as aggregate liquidity and productivity shocks, respectively. The  $M$ -by-1 vectors  $\mathbf{b}$  and  $\mathbf{z}$  denote the factor loadings, and  $\mathbf{m}$  the aggregate shocks into each industry's liquidity and productivity shocks. Together,  $\mathbf{b} F_t^B$  and  $\mathbf{z} F_t^Z$  comprise the aggregate components of  $\mathbf{B}_t$  and  $\mathbf{z}_t$ .

The residual components,  $\mathbf{u}_t$  and  $\mathbf{v}_t$ , unexplained by the common factors, are the idiosyncratic or industry-level shocks affecting each industry's liquidity and productivity growth. I assume that  $F_t^B$ ;  $\mathbf{u}_t$  and  $F_t^Z$ ;  $\mathbf{v}_t$  are each serially uncorrelated,  $F_t^B$ ;  $\mathbf{u}_t$ ;  $F_t^Z$ ; and  $\mathbf{v}_t$  are mutually uncorrelated, and the variance-covariance matrices of  $\mathbf{u}_t$  and  $\mathbf{v}_t$ ,  $\Sigma_u$  and  $\Sigma_v$ , are diagonal.

I assume further that the factors follow an AR(1) process such that

$$F_t^B = \rho_B F_{t-1}^B + \epsilon_t^B$$

$$F_t^Z = \rho_Z F_{t-1}^Z + \epsilon_t^Z$$

Here,  $\epsilon_t^B$  and  $\epsilon_t^Z$  are independently and identically distributed. Hence, I have two dynamic factor models; one for the liquidity shocks  $\mathbf{B}_t$  and one for the productivity shocks  $\mathbf{z}_t$ .

I use standard methods to estimate the model. To predict the factors, I use both a one-step prediction method and Kalman smoother. The Kalman smoother yields factors which explain more of the data. Since it

Figure 7:

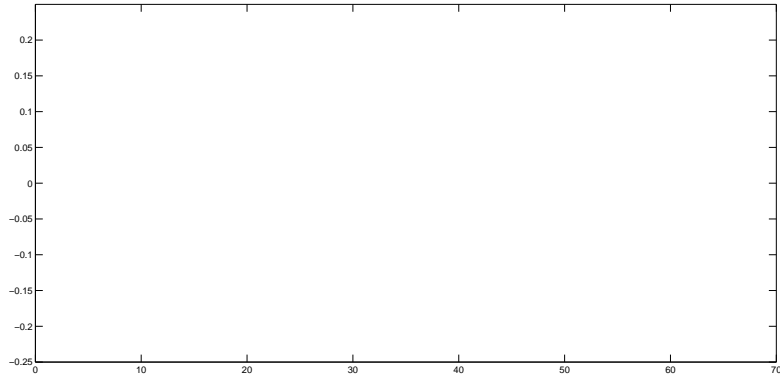
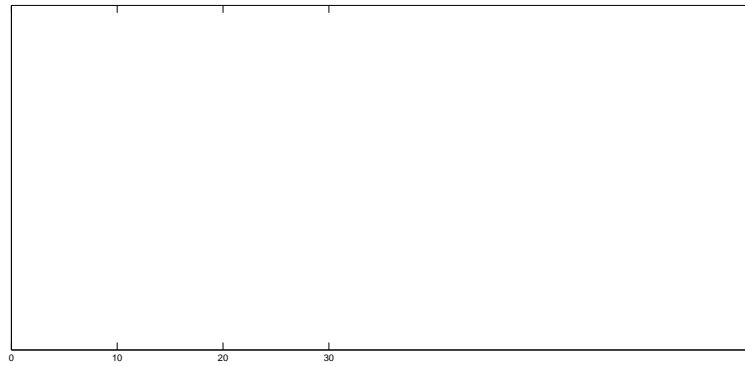


Figure 8:



## 7 Empirical Results

I now present and discuss the empirical results using the shocks estimated in the previous sections.

### 7.1 Aggregate Volatility Over Full Sample Period

In this section, I use the shocks estimated in the previous section to estimate how much of observed volatility in aggregate industrial production from 1997Q1:2013Q4 can be explained by each type of shock. In addition, I estimate the contribution of the credit network of the US industrial production industries to aggregate volatility. What follows is a brief summary of the procedure; a more detailed description is given in the Appendix.

#### 7.1.1 Shocks and Aggregate Fluctuations

Let the variance-covariance matrix of industry output growth  $X_t$  be denoted by  $\Sigma_{XX}$ . In addition, let  $s$  denote the  $M$ -by-1 vector of industry shares of aggregate output during the median year of my sample, 2005. Since these shares are close to constant across the quarters in my sample, the volatility of aggregate industrial output - henceforth **aggregate volatility** - can be approximated by  $\sigma^2$ , where

$$\sigma^2 = s' \Sigma_{XX} s$$

The factor model described above implies the following identities for the variance-covariance matrices of output growth  $X_t$  and those of the shocks  $B_t$  and  $Z_t$ .

$$\Sigma_{XX} = G_X B B' G_X' + H_X Z Z' H_X'$$

$$B B' = B B' + u u' \quad Z Z' = Z Z' + v v'$$

The fraction of observed aggregate volatility generated by aggregate liquidity shocks can be computed as the ratio of volatility generated by the aggregate component of  $B_t$  to  $\sigma^2$ .

$$\frac{s' G_X B B' G_X' s}{s' \Sigma_{XX} s}$$

Table 1: Composition of Agg. Vol.: 1997Q1:2013Q4

	Fraction of Agg. Vol. Explained
Productivity Shocks	.210
Agg. Component	.066
Idios. Component	.144
Liquidity Shocks	.790
Agg. Component	.654
Idios. Component	.136

The results of this analysis are summarized in Table (). I find that, for the full sample period 1997Q1:2013Q4, aggregate volatility in industrial production is about 0.19%<sup>12</sup>, and is driven primarily by idiosyncratic ro-

Table 2: Contribution of Credit Network

	Contribution of Credit Network
Effect of Prod. Shocks On Agg. Vol.	.019
Effect of Liq. Shocks On Agg. Vol.	.211
Total Agg. Volatility	.171

the theoretical analysis, the credit network primarily propagates liquidity shocks. Indeed, most of the effect of the credit network is in amplifying the aggregate liquidity shock.

### 7.1.3 Discussion

In summary, the main results of this analysis are that, when taking into account the credit linkages between industries,

1. Aggregate productivity shocks **do not** play an important role in aggregate fluctuations in industrial production
2. Aggregate volatility is driven primarily by **idiosyncratic productivity** shocks and **aggregate liquidity** shocks
3. The credit network of the economy plays an important role in amplifying fluctuations in aggregate output

How does this compare to the findings of studies? Foerster et al. (2011) show that, when accounting for the effects of in-out-out linkages in propagating shocks across industries, the role of aggregate productivity shocks in driving the business cycle is diminished; more of aggregate volatility in IP can be explained by industry-level productivity shocks. Nevertheless, they still find a quantitatively large role for aggregate productivity shocks. On the other hand, my analysis shows that when one takes into account the credit linkages between non-financial firms in the economy, the role of aggregate productivity shocks is minimal. On the contrary, aggregate **liquidity** shocks seem to play a vital role the business cycle. Indeed, the importance of shocks emanating from the financial sector to real economy as a whole is well-documented.

## 7.2 Great Recession



## A1. Agency Problem

—

## A2. Simple Model Solution

Solved in closed-form recursively, starting with the final firm in the chain, firm M.

### Firm M

Recall that firm M collects none of its sales from the household up front (does not give the household any trade credit,  $\tau_M=0$ ). Then its problem is to choose its input purchases, loan from the bank, and the trade credit loan from M-1, to maximize its profits, subject to its cash-in-advance, supplier borrowing, and bank borrowing constraints.

$$\max_{n_M; x_{M-1}; b_M; M-1} p_M x_M - w n_M - p_{M-1} x_{M-1}$$

$$s.t: w n_i + (1 - \tau_{i-1}) p_{i-1} x_{i-1} = b_i + \tau_{i-1} p_{i-1} x_{i-1} + p_M x_M - M$$

$$b_M = (B_M + (1 - \tau_M) p_M x_M)$$

$$M = p_{M-1} x_{M-1} - M; M-1 p_{M-1} x_{M-1}$$

Recall that the firm does not collect any cash-in-advance from the household, so that its trade credit  $M = p_M x_M$ . Also recall that its borrowing constraints ( ) and ( ) bind in equilibrium, so that the problem can be rewritten

$$\max_{n_M; x_{M-1}; M} p_M x_M - w n_M - p_{M-1} x_{M-1}$$

$$s.t: w n_M + p_{M-1} x_{M-1} = M - p_M x_M$$

where

$$M = M; M-1 + B_M$$

Notice that because  $M = p_M x_M$ ,  $M$  is given by exogenous parameters.



$$W = M^M$$

$$M^{-1} = (M;M^{-1} + B_M + 1) \frac{M p_M x_M}{p_M^{-1} x_M^{-1}}$$

And () and () imply that  $\frac{p_M x_M}{p_M^{-1} x_M^{-1}} = \frac{1}{M! M;M^{-1}(1 - M)}$ . Therefore,

$$M^{-1} = (M;M^{-1} + B_M + 1) \frac{M}{M! M;M^{-1}(1 - M)}$$

$$w n_{M-1} = \dots$$

Continuing recursively, we can write  $n_i$  as a function of  $x_M$ , for each  $i$  (LEFT OFF HERE)

$$w n_i = p_M x_M \prod_{j=i}^M A_j \dots$$

The household's preferences and optimality conditions imply

$$w = \frac{V'(N)}{U'(x_M)} = x_M$$

Let good  $M$  be the numeraire. Combining ( ) with ( ) yields a closed-form expression for each firm's labor use.

$$n_i = \dots$$

Recall that the production functions imply that aggregate output can be written

Then ( ) and ( ) yield a closed-form expression for aggregate output.

### A3. Production Influence Vector

$$v = \begin{pmatrix} v_1 & v_2 & v_3 & \dots & v_M \\ 0 & v_1 & v_2 & \dots & v_{M-1} \\ 0 & 0 & v_1 & \dots & v_{M-2} \\ \vdots & & & \ddots & \vdots \\ 0 & 0 & 0 & \dots & v_1 \end{pmatrix}$$

$v_i$  captures downstream propagation (supply effects). But misses upstream demand effects. Total effect is  $\sum_{j=1}^i v_j$

## A4. Proof of Proposition 1

Proof: From ( ) (chi definition) and ( ) (phi interdependence),

$$r_i = \min \left\{ 1; \frac{1}{r_i} B_i + \frac{1}{i! i! (1 - r_i)} \right\}$$

It follows that

$$\frac{d r_i}{d B_i} = \begin{cases} < \frac{1}{r_i} > 0 & \text{if } r_i < 1 \\ 0 & \text{otherwise} \end{cases}$$

$$\frac{d r_j}{d B_i} = 0 \quad \text{if } j > i \quad \text{and} \quad \frac{d r_j}{d B_i} = \frac{1}{r_i} > 0 \quad \text{for } j = i$$

Putting these cases together, we can write  $\frac{d \log r_j}{d B_i}$  for any  $j$ .

$$\frac{d \log r_j}{d B_i} = \begin{cases} \frac{1}{r_i} > 0 & \text{if } j = i \\ \frac{1}{r_j} \frac{1}{i! i! (1 - r_i)} \frac{d r_i}{d B_i} > 0 & \text{if } j < i \\ 0 & \text{otherwise} \end{cases}$$

It follows that  $\frac{d \log r_j}{d B_i} \geq 0$  and  $\frac{d}{d r_j} \frac{d \log r_j}{d B_i} \geq 0$ .

## A5. Solution Procedure in General Model

Claim: solution procedure takes same form in general model as in stylized.

Firm  $i$ 's problem is to maximize profits subject to its liquidity constraint.

$$\max_{n_i; f; x_{is}; g_{s1}} p_i x_i - w n_i - \sum_{s=1}^M p_s x_{is}$$

$$w n_i + \sum_{s=1}^M p_s x_{is} \leq \bar{y}_i p_i x_i$$

where  $\bar{y}_i$  denotes the tightness of  $i$ 's liquidity constraint.



—  
 Use of I-O tables and Comstat data  
 —

## A6. Log-Linearized System

Stars are point around which system is approximated. Calibrated equilibrium values.

For all  $i$  and  $j$

In order: firm  $i$ 's optimality condition for input  $j$ , firm  $i$ 's optimality condition for labor, definition of wedge  $h_i$ , household optimality condition for consumption of each good, market clearing for good  $i$ , production function for firm  $i$ , household budget constraint, labor market clearing condition, household optimality for labor versus aggregate consumption.

$$\begin{aligned}
 p_j + x_{ij} &= \tilde{r}_i + p_i + x_i & w + r_i &= \tilde{r}_i + p_i + x_i & \tilde{r}_i &= \begin{cases} \delta & \text{if } i < 1 \\ 0 & \text{if } i \geq 1 \end{cases}
 \end{aligned}$$



Log-linearizing  $\tilde{r}_i$  yields

$$\tilde{r}_i = \begin{cases} \frac{\beta}{1-\beta} \frac{B_i}{r_i} \tilde{B}_i + \frac{P}{r_i} \sum_{c=1}^M \frac{c^i}{(1-c)^{i+1}} \tilde{c} & \text{if } i < 1 \\ 0 & \text{otherwise} \end{cases}$$

Thus, in the full model wedges  $\tilde{r}_i$  respond endogenously to direct liquidity shocks  $\tilde{B}_i$  and to changes in its customers' wedges  $\tilde{c}$ .

$$\tilde{\gamma}_i = \begin{cases} < \tilde{\gamma}_i^c & \text{if } \gamma_i < 1 \\ 0 & \text{otherwise} \end{cases}$$

where

$$\tilde{\gamma}_i^c = \frac{\mathbf{B}_i}{r_{i,i}} \mathbf{B}_i + \frac{\mathbf{X}^M}{r_{i,i}} \sum_{c=1}^M \frac{c_i}{c(1-c)^{c-1}} \tilde{\gamma}_c - \frac{\mathbf{X}^M}{r_{i,i}} \sum_{c=1}^M \frac{c_i}{c(1-c)^{c-1}} \tilde{\gamma}_{ci}$$

and

$$\tilde{\gamma}_{ci} = \mathbf{X}_{ci} \mathbf{X}_i$$

This expression says that industry  $i$ 's wedge can change either from direct liquidity shock to  $i$  (given by  $\mathbf{B}_i$ ), changes in the wedges of customers (given by  $\tilde{\gamma}_i$ ) through credit linkages  $c_i$ , or changes in the composition of industry  $i$ 's sales (given by  $\tilde{\gamma}_{ci}$  for all customers  $c$ ), also through credit linkages.

Consider first a liquidity shock to industry  $j$ , given by  $\mathbf{B}_j < 0$ . How does this affect  $\gamma_i$ , and how does this effect depend on  $i$ 's credit linkages with  $j$ ? From (1), we can see that there are two effects. First, the shock reduces  $\gamma_j$ , so that  $\tilde{\gamma}_j < 0$ . This pushes  $\gamma_i$  down. Second, because  $i$  has  $M$  customers,  $\mathbf{X}_{ji}$  falls by more than  $\mathbf{X}_j$  falls. Therefore,  $j$ 's share of  $i$ 's output  $\gamma_{ji}$  falls, and  $\tilde{\gamma}_{ji} < 0$ . This pushes  $\gamma_i$  up. The stronger is  $j$

small, as discussed in the quantitative analysis.

–

## A9. Aggregate Volatility

Recall that the growth in industry output can be written as a function of the industry liquidity and productivity shocks. Recall that  $X_t$  is a vector of the percentage change  $x_{it}$  in each industry's output at time  $t$ .

$$X_t = G_X B_t + H_X z_t$$

And the shocks  $B_t$  and  $z_t$ , in turn, are composed of an aggregate and idiosyncratic components.

$$B_t = B F_t^B + u_t \quad F_t^B = B F_{t-1}^B + \epsilon_t^B$$

$$z_t = z F_t^z + v_t \quad F_t^z = z F_{t-1}^z + \epsilon_t^z$$

Then letting  $\Sigma_X$  denote the variance-covariance matrix of  $X_t$  (and similarly for the other variables), we have

$$\Sigma_X = G_X \Sigma_B G_X^0 + H_X \Sigma_z H_X^0$$

$$\Sigma_B = B \Sigma_{FF}^B + \Sigma_{uu}$$

$$\Sigma_z = z \Sigma_{FF}^z + \Sigma_{vv}$$

where  $\Sigma_{uu}$  and  $\Sigma_{vv}$  are diagonal matrices.

Aggregate manufacturing output at time  $t$  is defined as  $\sum_i x_{it}$ . Let



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