Ranking Ranking Rules

Abstract: Transitivity is a fundamental requirement for consistency. Legal systems, especially when composed over time and by different agencies, may encounter non-transitive cycles, in which by one rule the law prefers one outcome a over another outcome b, by another rule b trumps some third result c, but a third rule ranks c higher than a. This paper discusses a new solution to such cycles in which the relevant rules of preferences are ranked and then applied until a transitive order of the options is obtained. The paper provides a formal generalization of this solution, and demonstrates its possible implementation to some legal issues. It is also shown that this solution can be traced to the Rabbinic

There are several theoretical objections to violations of transitivity. The first relates to the requirement that judges use logical reasoning in their decisions (see Levi, 1949; Dworkin, 1986). If three rules create a cycle as above and a dispute between a and c needs to be resolved, then the logical conclusion that ranks a higher than c (since one law ranks a above b, and another law ranks b higher than c) clashes with the direct ruling concerning these two options, leaving the judge with contradicting instructions. Transitivity is also a key axiom in choice theory (Harsanyi, 1955; Sen, 1986; Kreps, 1988). The best support for transitivity in this context comes from the requirement that choice should be consistent (see, e.g. Sugden, 1985; Rubinstein, 2006). Let X be a set of alternatives, and let C be a choice function defined on all nonempty subsets of X. Suppose that for three alternatives a, b, and c, the choice function C satisfies $C(a,b) = a$, $C(b,c) = c$, but $C(a,c) = c$. What should $C(a,b,c)$ be? If the outcome of C is a singleton (say a), then there is another outcome that is even better. And if the outcome of $C(a,b,c)$ is not a singleton, then one of these outcomes is dominated by the other.

Sometimes, an implied assumption of the modern discussion of non-transitive cycles is that the rules or preferences that lead to this outcome enjoy a similar normative status. Consequently, an arbitrary agenda setting is inevitable to solve the cycle (for example, Levmore, 2005); and specifically in the context of collegial courts (Kornhauser, 1992; List and Pettit, 2002). The solution we offer points to the fact that some external set of norms may well provide a ranking of the ranking rules. This ranking, together with the logic of transitivity, may provide a solution to cycles. As we show in Section 3, this latter solution can be traced to medieval Rabbinic thought and is new to the literature concerning choice.

the ranking rule says nothing about the relationship between elements of B and C

ranked highest, and then we can have either $d \succ a \succ b$ or $d \succ b \succ a$, or b is ranked highest, in which case $b \succ d \succ a$. □

Sometimes, given higher ranked rules, the added information contradicts itself and must therefore be ignored. This is the case in the next example.

Example 4: $X = \{a, b, c, d\}$. There are three rules: $a \succ^1 b$, $c \succ^2 d$, and $\{b, d\} \succ^3 \{a, c\}$. The third rule implies, as before, $b \succ^3 a$, $b \succ^3 c$, $d \succ^3 a$, and $d \succ^3 c$. But if we rank the rules 1-2-3, then we have to ignore $b \succ^3 a$ and $d \succ^3 c$. We are thus left with $b \succ^3 c$ and $d \succ^3 a$.

Given the ranking 1-2-3, if we use the third rule to obtain $b \succ c$, then we must deduce, using transitivity that $a \succ b \succ c \succ d$. However, if we use the third rule to obtain that $d \succ a$, then transitivity implies $c \succ d \succ a \succ b$. The two possible extensions contradict each other. As we don't have a reason to prefer the rule $b \succ c$ to $d \succ a$, we will ignore both. All we can therefore say in this case is that $a \succ b$ and $c \succ d$. □

Even when some information that a rule provides must be ignored, part of the information should be usded. Consider the following case.

Example 5: $X = \{a, b, c, d, e\}$. There are three rules: $a \succ^1 b$, $c \succ^2 d$, and $\{b, d, e\} \succ^3 \{a, c\}$, and the rules are ranked 1-2-3. Similar to the previous example, we cannot use either $b \succ^3 c$ or $d \succ^3 a$. But this time rule 3 is not entirely useless as nothing prevents us from using the information $e \succ^3 a$ and $e \succ^3 c$. By rules 1 and 2 and by transitivity, we thus obtain $e \succ a \succ b$ and $e \succ c \succ d$. □

In general, when it is the turn of rule i to be used, we will first eliminate all its pairwise comparisons that are already contradicted by previous rules, or by transitive conclusions obtained from these rules. We then include all the pairwise comparisons obtained from rule i, provided they do not lead to contradictions with other rule i comparisons or with their conclusions. In the Appendix we offer a complete algorithm for our method for the case where all rules are of the form $a \succ b$ and all pairs (there are $\frac{1}{2}n(n-1)$ of them) are thus compared. We show that in this case our algorithm yields a complete and transitive ranking of the n options.

Our method should not be confused with lexicographic preferences. Tversky (1972) suggested the following ranking rule: There are k (ordered) criteria. Option a is ranked higher than b if the first criterion in which a and b are not indifferent ranks a above b. This idea was expanded by Manzini and Mariotti (2007, 2010) to include sequential (or inductive) elimination, where at step i a relation P_i is applied to the set M_{i-1} of all options that survived the previous step, and only options that are not dominated in P_i by any other option in M_{i-1} survive to the next stage (this new set is labeled M_i).

Example 1 illustrates why the procedure we suggest is different from the above elimination mechanisms. In this example, $X = \{a, b, c\}$, $a \succ^1 b$, $b \succ^2 c$, decision in two (or more) different legal disputes. For instance, in a typical contract law case, the proposition whether the defendant has breached the contract is a compound of the atomic decisions whether there was a contract at all, and whether there was an act constituting breach of such contract. Assume that two out of the three judges decide in favor of the defendant, but one of them decides so since she finds there was no contract (but if there were one, this judge believes it was breached), while the other judge is in the opposite position in both issues (there was a contract but it was not breached). The minority judge rules that there was a contract and it was breached. Aggregating these atomic positions separately results in ruling in favor of the plaintiff. This problem is often resolved by employing a decision-rule according to which the outcome is based on aggregating the judges' compound, rather than atomic decisions. In contrast, we analyze a different case. We refer to instances in which there is no dispute regarding each of the atomic positions, that is, regarding the ranking of each of the given pairs of possible outcomes. The problem we address stems from-37The t14gchions,posihereo6(no3-378-(op31(thwe)-se(re the texts. The example discussed below may seem contrived. Indeed it is. The

There may be another relevant criterion, about which the Mishnah is silent. In general, the gathering of a group of various offerings may occur in two different circumstances:

(i) Several offerings accumulated at random, as one person or more brought them simultaneously, for different ritual purposes.

(ii) A group of offerings is brought as one sacrificial unit for a specific ritual. For example, the pair of a bird sin-offering and a lamb burnt-offering, brought by a mother after giving birth.[8]

The Mishnah clearly intends to impose its ruling in the first situation – it ranks three rules are applied in the following way: If two or more offerings accumulate at random, then they are sacrificed according to Rules 1 and 2. But if a set of offerings is brought as one sacrificial unit, then Rule 3 applies. It is only in the latter case that a bird sin-offering precedes a cattle burnt-offering.[12]

Yet this does not supply us with instructions for all possible events. What if situations (i) and (ii) occur simultaneously? Consider, for example, the following case. A person arrives at the temple with a tithe (a); at the same time a woman after giving birth brings a sacrificial pair of animals: a bird for sin-offering (b) and a lamb for burnt-offering (c). The internal order of the group $\{b, c\}$ is governed by Rule 3: $b \succ c$. But which should be brought first to the altar, the tithe a or the woman's offerings b and c? On the one hand, the woman's pair contains c, a burnt-offering (the lamb), and according to Rule 2, $c \succ a$. On the other hand, the actual choice is between a, the tithe, and b, the bird, and according to Rule 1, $a \succ b$. Apparently, this is a simple ranking problem of two items, where one of the items is composed of two animals, and the above two rankings are the only ways in which it can be answered. This is most probably the original form of the question asked by the Talmud (Zev. 90b): "A bird sin-offering and an animal burnt-offering and tithe – which of these come first?" meaning, "[a pair of] {a bird sin-offering and an animal burnt-offering} and tithe – which of these come first?" The Talmud offers two answers:

1. Here [= in Babylon] they held that an animal offering is superior [hence the tithe comes first];
2. In the West [= Palestine] they say: The superiority of the animal burnt-offering over tithe enters in the bird sin-offering and elevates it over the tithe [hence the bird sin-offering comes first].

The Babylonian solution[13]

adapt the answer to its new role. From this hermeneutic process, the original simple and intuitive solution came out as a novel strategy for breaking non-transitive cycles.[17]

4 Application to current legal dilemmas

In what follows, we illustrate possible implementations of our analysis to actual legal issues. We discuss three examples: conflict of title between an original owner, an intermediate purchaser, and a good-faith purchaser for value; the conflict between the "disparate-treatment" and "disparate-impact" doctrines of anti-discrimination laws; and the right of beneficiaries to recover from tortfeasors.

4.1 Conflict of title

Conflict of title rights in an asset between a first-in-time claimant (the "original owner") and a good-faith purchaser for value provides an example to the possibility of non-transitive cycles. These conflicts arise when parties not in contractual privity assert simultaneous claims of rights over the same asset whose concurrent discharge is legally impossible, and the law is called upon to resolve the conflict.

The original owner, a, and the good-faith purchaser, c, are linked through the activities of an intermediate seller, b, who transacted with each of them. The

If this cycle is not resolved, the ultimate result depends on the decision

Title VII of the Civil Rights Act of 1964 contains two central anti-discrimination doctrines – the so-called "disparate-treatment" and "disparate-impact" prohibitions. The former norm proscribes an employer the use of an employment practice there are other possible practices that are similarly job related (for instance, in terms of the predictive power of a qualification test regarding an employee's performance) and can be expected to have a more equal impact. In short:

Rule 1: An employer is required to discard an imbalanced promotion test in favor of an alternative, which is as job related as the former or at least close to it, and whose result is more racially balanced. However,

Rule 2: An employer is not allowed to discard a suggested test on the basis of its racially imbalanced impact in favor of an alternative test, which is not as job related as the former or at least sufficiently close to it.

These two rules seem to provide a coherent solution. However, determining the legal outcome in a specific case is more complicated once employees' reliance interest is taken into account. Rules 1 and 2 deal with instances in which discarding a specific promotion test does not infringe legitimate expectations of the employees, either since the test was not yet implemented or the change of the test will affect only future decisions. But what if discarding the promotion test would infringe the employees' legitimate expectations and reliance interests? In circumstances in which an employment practice (such as a promotion test) has already been applied, the expectation and reliance interests of the employees who are eligible to some benefit according to this practice impose a constraint on the employer' *a* A promotion test which is highly job relevant, but has an imbalanced impact.

b A promotion test which is only moderately job relevant, but its impact is more racially balanced.

c A promotion test which is moderately job relevant, its impact is imbalanced, but it was already relied upon by certain employees.

Suppose a court is asked to compare test *a* with *b*. Both are yet to be given, from an interpretation of this provision). This rule applies to the pair $\{b,c\}$ only, and according to it, $b \succ c$.

These two stages yield that $c \succ a$ and $b \succ c$. Transitivity then implies $b \succ a$. It is true that had the employer implemented the promotional test a, it would not have been legitimate, due to Rule 2, to discard a in favor of b. However, given the assumed precedence of Rule 3, when c is the already selected and relied upon test, and the precedence of Rule 1 over Rule 2, the ranking of a and b by Rule 2 should be ignored in order to avoid intransitivity.[24]

The essential decision is therefore that of ranking the relevant rules. Seemingly, a rule set forth by Congress (Rules 1 and 2 in our case) should be applied before a judge-made rule (such as Rule 3 here). However, substantive reasons may well be relevant too, and these may lead to alternative rankings of the rules. For instance, requiring an individual to bear the burden of a loss – other than a mere lost benefit – to prevent imbalanced outcome may be considered as a form of taking, in violation of the Fifth Amendment,[25] such that Rule 3 takes precedence over the two other rules.

Setting the ranking of the rules will often be based on a normative theory which is outside the rules themselves and the ranking of the options that they yield. It is beyond the scope of this paper to resolve the dilemma what is the correct priority among the above three rules, or what is the general theory that determines the ranking of rules. Our point is that ranking the rules will enable courts to avoid cycles. Indeed, ranking the rules is actually the central role of courts in this regard.

4.3 Beneficiaries' right to recovery from Tortfeasors

The legal-system's commitment to avoid cycles can serve as a resourceful interpretation tool. In this subsection we comment on such possible uses of the solution concept that we offer here.

Assume that a judge should resolve a dispute between outcomes a and b, and that the relevant rule (Rule 1) prioritizes a over b. In principle, the opposite

24 Note that in our example although c wins in the first comparison (where Rule 3 is applied for the pair {a,c}), c is not the best policy overall as it is defeated by b.

25 See, e.g., Local 28 of the Sheet Metal Workers, Int'l Ass'n v. EEOC, 478 U.S. 421, 479 (1986) (upholding an affirmative action plan in part because it "did not require any member of the unionunion ranking of these two outcomes can be justified by invoking some third outcome c, and applying the governing norms regarding the trio, a, b, and c. If some other rule (Rule 2) determines that $b \succ c$, and a third one (Rule 3) results in $c \succ a$

cost of medical care furnished, and thus the injured person may be bound by a compromise agreement with the injured but the government isn't (e.g. U.S. v. Greene 266 F.Supp 976 (N.D.Ill. 1967); Holbrook v. Andersen Corp. 996 F.2d 1339 (3d.Cir. 1993). As a result, the injured person, a, is entitled to compensation from the government, b, for certain expenses, such that $a \succ b$ (Rule 1). Similarly, the government is entitled to recovery from the injurer, c, such that $b \succ c$ (Rule 2). However, c is entitled to recover from a (based, for instance, on their prior settlement agreement), such that $c \succ a$ (Rule 3). Nevertheless, a non-transitive cycle does not occur, since a is not entitled to recover again from b. Therefore, even if Rule 3 is of a higher normative ranking than the other rules, there is no basis for overriding Rules 1 or 2.

We thus suggest that the transitivity argument should be used only when all elements (three or more) of a group should be ranked. If each pair of claims can be ranked independently of the other outcomes and there is no natural reason to rank all claims simultaneously, one cannot justifiably refer to the requirement of

Finally, the fact that the scope of rules that rank specific laws is typically wider than the scope of specific laws establishes our claim.[27]

Appendix: ranking rules

This appendix explains how to create a complete ranking from ordered pairwise comparisons.

Let $N = \{1, \ldots, n\}$ and let c be a choice function such that for all $i \neq j$, $c(i,j)$ exists and is a singleton. Let R be a linear ranking of the $n(n-1)/2$ pairs $\{i,j\}$, $i \neq j$, that is,

$$\{i_1, j_1\} R \{i_2, j_2\} R \ldots R \{i_{n(n-1)/2}, j_{n(n-1)/2}\}$$

and for no $k > 1$, $\{i_k, j_k\} R \{i_{k-1}, j_{k-1}\}$. Let $\varnothing \neq T \subseteq N$ and construct inductively partial linear orders $\succ_0^T, \ldots, \succ_{n(n-1)/2}^T$ on T as follows:

1. $\succ_0^T = \varnothing$ (that is, no elements of T are compared by $\succ_0^T$). Set $k = 1$ and move to the next step.
2. If $k = \frac{n(n-1)}{2} + 1$ set $\succ^T = \succ_{n(n-1)/2}^T$ and the construction is complete. Otherwise, move to the next step.
3. If $\{i_k, j_k\} * T$, set $\succ_k^T = \succ_{k-1}^T$, increase the value of k by 1, and move to step 2. Otherwise, move to the next step.
4. If i_k and j_k are comparable by $\succ_{k-1}^T$, set $\succ_k^T = \succ_{k-1}^T$,[28] increase the value of k by 1, and move to step 2. Otherwise, move to the next step.
5. Define $>_k^T$ as follows:
   - For all $\{i,j\} \neq \{i_k, j_k$

This method may not work if we drop the requirement that for no $k > 1$, $\{i_k, j_k\} R \{i_{k-1}, j_{k-1}\}$. Let $N = \{a_1, \ldots, a_4\}$, and suppose that $c(a_1, a_2) = a_1$, $c(a_2, a_3) = a_2$, $c(a_3, a_4) = a_3$, and $c(a_1, a_4) = a_4$. Suppose further that the top four pairs are $\{a_1, a_2\}R\{a_3, a_4\}R\{a_2, a_3\}R\{a_1, a_4\}$ but also $\{a_1, a_4\}R\{a_2, a_3\}$. The two orders $a_1 \succ a_2 \succ a_3 \succ a_4$ and $a_3 \succ a_4 \succ a_1 \succ a_2$ are consistent with the above algorithm.

Acknowledgment:

\ _, . 2003.