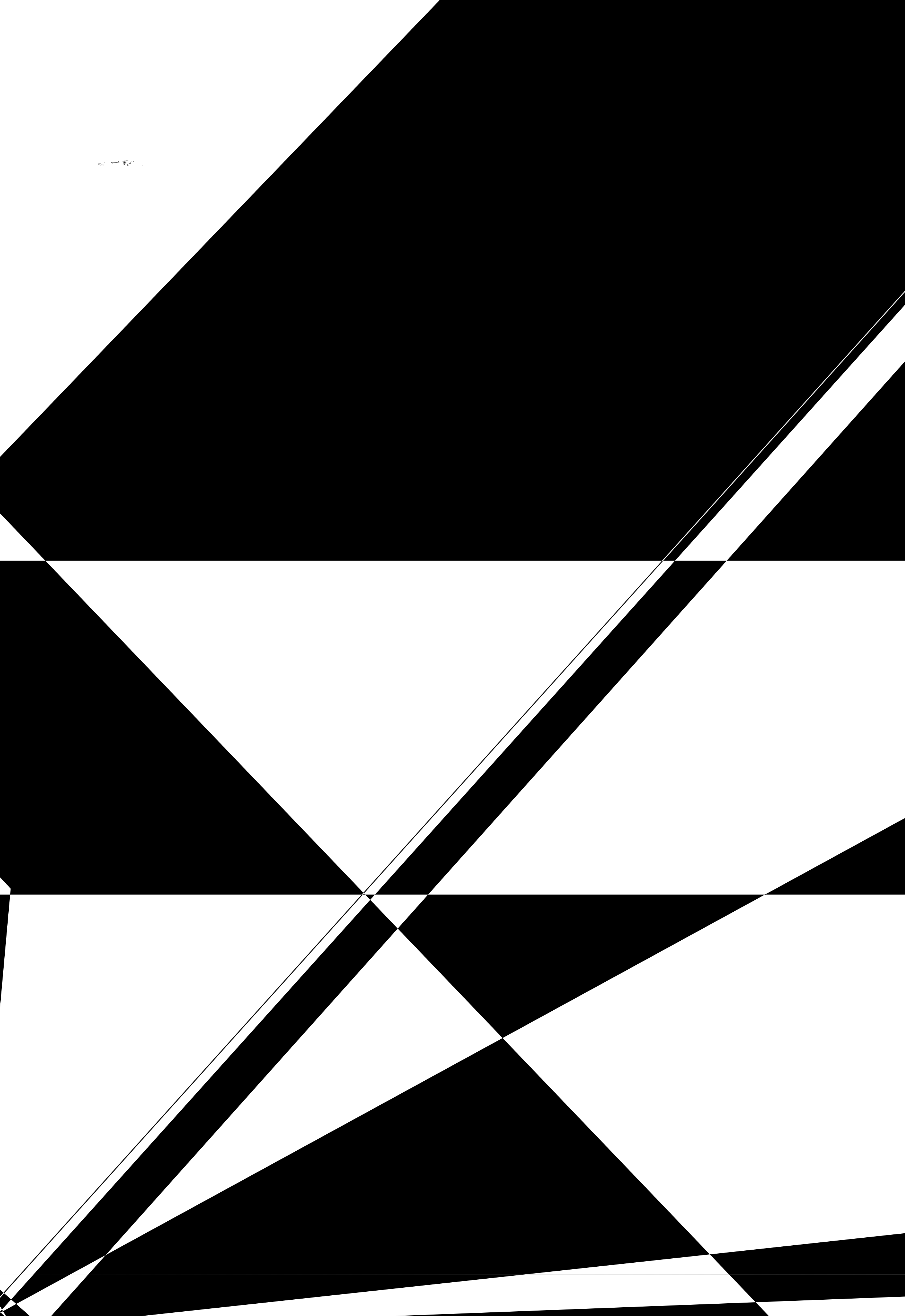

$X = (x_1, s_1; \ldots; x_n$

[illegible] (3) [illegible]

[illegible] x [illegible] X [illegible] Y, [illegible] Y. [illegible] Y [illegible] Y' [illegible] x [illegible]

Lemma 1. *Let $\succeq$ be a distribution-regret preference relation. Then $\succeq$ admits a two-dimensional regret function ${}^*: \mathcal{D} \times \mathcal{D} \to \Re$ and a regret functional V^* such that*

$$\begin{aligned} X \succeq Y \quad &\Longleftrightarrow \quad V^*\big({}^*(x_1, c_Y), p_1; \ldots; {}^*(x_n, c_Y), p_n\big) \geqslant 0 \\ &\Longleftrightarrow \quad V^*\big({}^*(y_1, c_X), q_1; \ldots; {}^*(y_m, c_X), q_m\big) \leqslant 0 \end{aligned}$$

where c_X and c_Y are the certainty equivalents of X and Y respectively.

Proof. [illegible] V [illegible] $\succeq$. [illegible] $y \in \mathcal{D}$, [illegible] ${}^*(x, y) = (x, \delta_y)$, [illegible] δ_y [illegible] y [illegible] 1, [illegible]

$$V^*\big({}^*(x_1, y), p_1; \ldots; {}^*(x_n, y), p_n\big) = V\big((x_1, \delta_y), p_1; \ldots; (x_n, \delta_y), p_n\big)$$

[illegible]

$$\begin{aligned} X \succeq Y \quad &\Longleftrightarrow \quad X \succeq \delta_{c_Y} \\ &\Longleftrightarrow \quad V\big((x_1, \delta_{c_Y}), p_1; \ldots; (x_n, \delta_{c_Y}), p_n\big) \geqslant 0 \\ &\Longleftrightarrow \quad V^*\big({}^*(x_1, c_Y), p_1; \ldots; {}^*(x_n, c_Y), p_n\big) \geqslant 0 \qquad \square \end{aligned}$$

[illegible] 1

1,

Definition 4. $\succeq$ *distribution-regret based*

$: \mathcal{D} \times \mathcal{D} \rightarrow \Re$,

$V : \rightarrow \Re$

$$X \succeq Y \quad \textit{iff} \quad V\big(\Psi(X, c_Y)\big) \geqslant 0 \quad \textit{iff} \quad 0 \geqslant V\big(\Psi(Y, c_X)\big),$$

$\Psi(X, c_Y) = (\ (x_1, c_Y), p_1; \ldots; \ (x_n, c_Y), p_n)$ X δ_{c_Y} (Y), $\Psi(Y, c_X)$ Y δ_{c_X} (X),

x

$$X \sim \delta_{c_Y} \quad \Longrightarrow \quad V\Big(\; \big(n \, d \qquad V$$

Proposition 3. *If the preference relation $\succeq$ is consistent then it satisfies distribution regret.*

Proof. $Z \in \mathcal{L}$ ($Z = \delta_x$ x $\mathcal{D}$), $\lambda(Z)$ λ $f(c_Z, \lambda) = 0$.[10] 5, $\lambda(Z)$ $Z \in \mathcal{L}$, $Z = \delta_x$ x $\mathcal{D}$. U $\succeq$ $U(\delta_x) = x$ x. $(x, c_Y) = f(x, \lambda(Y))$.[11] (x, c_Y) x f $\lambda(Y)$ $\lambda(Y)$ Y 0. $\succeq$ $\lambda(Z) = -c_Z$ $(x, y) = x - y$. 5, $x \in \mathcal{D}$ $f(x, \lambda(Y)) \in \mathcal{D}$.

$$
\begin{aligned}
f\big(X, \lambda(Y)\big) &= \big(f\big(x_1, \lambda(Y)\big), p_1; \ldots; f\big(x_n, \lambda(Y)\big), p_n\big) \\
&= \big(\quad (x_1, c_Y), p_1; \ldots; \quad (x_n, c_Y), p_n\big) \\
&= \Psi(X, c_Y)
\end{aligned} \tag{4}
$$

$$
\begin{aligned}
X \succeq Y \sim \delta_{c_Y} \quad &\Longleftrightarrow \quad f\big(X, \lambda(Y)\big) \succeq f\big(Y, \lambda(Y)\big) \sim \delta_{f(c_Y, \lambda(Y))} = \delta_0 \\
&\Longleftrightarrow \quad U\big(f\big(X, \lambda(Y)\big)\big) \geqslant U\big(f\big(Y, \lambda(Y)\big)\big) = U(\delta_0) = 0
\end{aligned}
$$

$$
V\big(\Psi(X, c_Y)\big) = U\big(f\big(X, \lambda(Y)\big)\big) = U\big(\Psi(X, c_Y)\big)
$$

((4)).

$$
\begin{aligned}
X \succeq Y \quad &\Longleftrightarrow \quad U\big(\Psi(X, c_Y)\big) \geqslant 0 \\
&\Longleftrightarrow \quad V\big(\Psi(X, c_Y)\big) \geqslant 0
\end{aligned}
$$

$\succeq$. □

(2). $u(0) = 0$. $Z = (z_1, r_1; \ldots; z_n, r_n)$ $z_1 \leqslant \ldots \leqslant z_n$, c_Z Z

$$
c_Z = u^{-1}\left(u(z_1)g(r_1) + \sum_{i=2}^{n} u(z_i)\left[g\left(\sum_{j=1}^{i} p_j\right) - g\left(\sum_{j=1}^{i-1} p_j\right)\right]\right)
$$

$f(x, \lambda) = u^{-1}(u(x) + \lambda)$

$$
f\big(c_Z, \lambda(Z)\big) = 0 \quad \Longrightarrow \quad \lambda(Z) = -u(z_1)g(r_1) - \sum_{i=2}^{n} u(z_i)\left[g\left(\sum_{j=1}^{i} p_j\right) - g\left(\sum_{j=1}^{i-1} p_j\right)\right]
$$

U $\succeq$ $U(\delta_x) = x$ $U(Z) = c_Z$ (x, c_Y), x X Y,

10 $0 \in$ $[\mathcal{D}]$. $d \in$ $[\mathcal{D}]$, $f(c_Z, \lambda(Z)) = d$ $(x, x) = d$.

11 $(x, x) =$ $(x, c_{\delta_x}) = f(x, \lambda(\delta_x)) = 0$.

$$(x, c_Y) = f\big(x, \lambda(Y)\big) = u^{-1}\big(u(x) + \lambda(Y)\big)$$

[illegible] u [illegible] x [illegible] X [illegible] Y [illegible] x [illegible] ([illegible]) [illegible] Y. [illegible] u [illegible]

Example 3. [illegible] $\succeq$ [illegible] $\mathcal{I}$ [illegible] [12] [illegible] [5].

[illegible] $X, Y, Z \in \mathcal{I}$. [illegible] $\succeq$ [illegible] $V(\Psi(X, c_Z)) = V(\Psi(Y, c_Z)) = 0$, [illegible] $\mathcal{I}$, [illegible] $\alpha \in (0, 1)$, $V(\Psi(\alpha X + (1-\alpha)Y, c_Z)) = 0$.

[illegible] Z' [illegible] $\mathcal{I}$ [illegible] X' [illegible] Y' [illegible]

$$\Psi\big(X', c_{Z'}\big) =$$

$$[X] + \alpha\mu_X^+ = (1+\alpha)\alpha \quad \Longrightarrow \quad \alpha = \frac{-(1-\mu_X^+) + \sqrt{(1-\mu_X^+)^2 + 4\ [X]}}{2} \qquad (\ )$$

1, ,

$$[X] > 0 \qquad (X < 0) > 0 \quad \Longrightarrow \quad X \succ \delta_{[X]} \qquad (\ )$$

, $\mu_X^+ > E[X] > 0$ $X \sim \delta_\alpha$, . ( ) $\alpha > E[X]$.
, $\succeq$. $-1 < s, t < 0$, λ_0 ,
$f(-1, \lambda_0) = s$ $f(t, \lambda_0) = 0$. $z \geqslant t$

$$\left[\left(z, \frac{1+t}{1+z}; -1, \frac{z-t}{1+z}\right)\right] = t, \qquad \left(z, \frac{1+t}{1+z}; -1, \frac{z-t}{1+z}\right) \sim (t, 1)$$

$$\left(f(z, \lambda_0), \frac{1+t}{1+z}; s, \frac{z-t}{1+z}\right) \sim (0, 1) \qquad (10)$$

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Proposition 4. *If the preference relation* $\succeq$ *satisfies distribution regret with a commutative regret function* , *then it is consistent.*

Proof. $d \in \Re$ $(x, x) = d$ $x \in \mathcal{D}$. (4 d .)
$f(x, \lambda) = y$ $(x, y) = d - \lambda$. , x λ,

$$\big(x, f(x, \lambda)\big) = d - \lambda \tag{13}$$

$X \succeq Y$, , $V(\Psi(X, c_Y)) \geqslant 0$. (13)

$$\big(c_X, f(c_X, \lambda)\big) = \big(c_Y, f(c_Y, \lambda)\big) = d - \lambda$$

(12) $(c_X, c_Y) = (f(c_X, \lambda), f(c_Y, \lambda))$,

$$\Psi(\delta_{c_X}, c_Y) = \Psi\big(\delta_{f(c_X,\lambda)}, f(c_Y, \lambda)\big)$$

, $\delta_{c_X} \succeq \delta_{c_Y}$, $\delta_{f(c_X,\lambda)} \succeq \delta_{f(c_Y,\lambda)}$.
, (13) (12)

$$(x_i, c_X) = \big(f(x_i, \lambda), f(c_X, \lambda)\big)$$

, $\Psi(X, c$

5. Discussion

[illegible] *only* [illegible] 10 [illegible] 14 .[13]