Analysis Qualifying Exam ______Spring 2011

Please answer all 6 problems from each section and show your work. Each problem is worth 30 points.

SECTION 1: REAL ANALYSIS

In your proofs, you may use any major theorem, except the fact you are trying to prove, or a mere variant of it. State clearly what theorems you use. Good luck.

1 Let $f_n : X \to \mathbb{R}$ be (X, μ) measurable functions such that $0 \le f_n \le 1$ and μ is a finite measure. Prove that if

$$\lim_{n\to\infty} \int_X f_n \, d\mu = \mu(X)$$

then $f_n \to 1$ a.e.

2 Let f be Lebesgue integrable on $(0, a)$ and $g(x) = \int_x^a t^{-1} f(t) dt$.
Prove that g is Lebesgue integrable on $(0, a)$ and $\int_0^a g(x)dx = \int_0^a f(x)dx$.

3 Let μ, ν be finite measure on (X, M) with $\nu << \mu$. Define $\lambda = \mu + \nu$ and $f = \frac{d\nu}{d\lambda}$.
Prove that $0 \le$

4 Let $s \in \mathbb{C}$ and consider

$$\sum_{n=1}^{\infty} n^{-s}.$$

a) Prove that the sum converges absolutely and uniformly in the set $\{s \mid \Re(s) \geq \delta\}$ for any $\delta > 1$.

b) Assume the result in part a). Then the sum defines an analytic function on $U := \{s \mid \Re(s) > 1\}$